prof. Petr Malý, materiály na moodle 2
zápočet: 2 testy, alespoň 20/30 b 1h 3 př.
testy přibližně 7.4. a 19.5.
zkouška: pís. část 7,5/15 b 3 př. } průměr z
ús. část 2 otázky } pís. a 2 úst.

1. Úvod

historické okénko, vývoj z pozorování elektřiny, magnetismu a dále z optiky

2. Základy vektorové analýzy

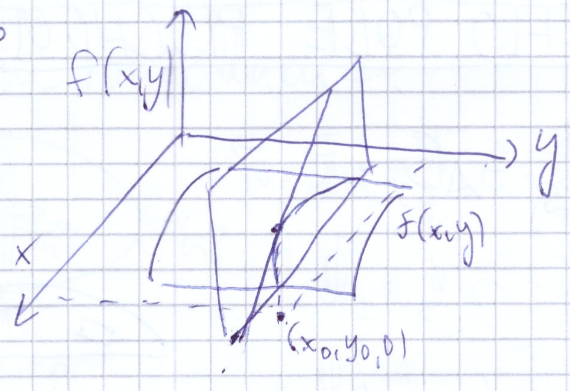
$f(x)$ ... fce jedné proměnné
$f(\underbrace{x,y,z}_{\vec{r}},t)$ ... fce více proměnných

pole = každému bodu prostoru přiřadím fční hodnotu

$\vec{a}(x,y,z,t)$ ... vektorová fce

$\left.\frac{df(x)}{dx}\right|_{x_0} = \lim_{h\to 0} \frac{f(x_0+h)-f(x_0)}{h}$ ... derivace podle x v x_0

$\left.\frac{\partial f(x,y,z)}{\partial x}\right|_{x_0,y_0,z_0} = \lim_{h\to 0} \frac{f(x_0+h,y_0,z_0)-f(x_0,y_0,z_0)}{h}$ ... parc. derivace

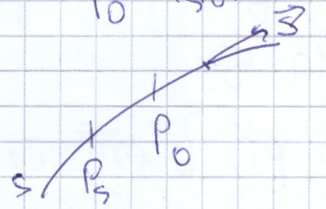


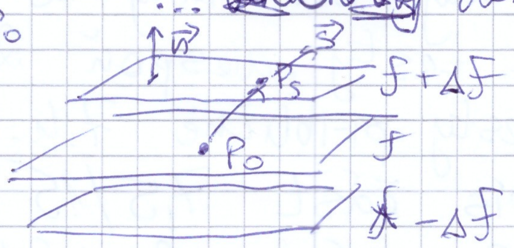
$\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$

$\frac{\partial^2 f}{\partial x^2}, \frac{\partial^2 f}{\partial x \partial y}$

GRADIENT

$$\left.\frac{\partial f(\vec{r})}{\partial s}\right|_{P_0} = \lim_{|P_sP_0|\to 0} \frac{f(P_s)-f(P_0)}{PP_0} \quad \dots \text{ derivace ve směru}$$

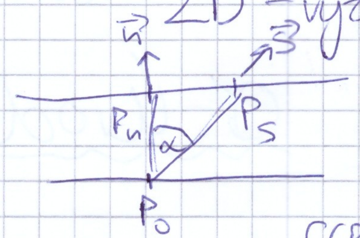




$\Delta f > 0$

1D

2D – význačný směr $\vec{n}$



$P_0P_n = P_0P_s \cos\alpha$

$$\lim_{P_sP_0\to 0} \frac{f(P_s)-f(P_0)}{PP_0} =$$

$$= \lim_{\substack{P_sP_0\to 0\\ P_nP_0\to 0}} \frac{f(P_n)-f(P_0)}{\frac{P_0P_n}{\cos\alpha}} = \cos\alpha \lim_{P_nP_0\to 0} \frac{f(P_n)-f(P_0)}{P_0P_n} = \cos\alpha \frac{\partial f}{\partial n}$$

pro $\vec{n}, \vec{s}$ jednotk. $\cos\alpha = \vec{n}\cdot\vec{s}$

$$\left.\frac{\partial f}{\partial s}\right|_{P_0} = \left.\frac{\partial f}{\partial n}\right|_{P_0} \cos\alpha = \underbrace{\frac{\partial f}{\partial n}\vec{n}}_{\text{grad } f}\cdot\vec{s}$$

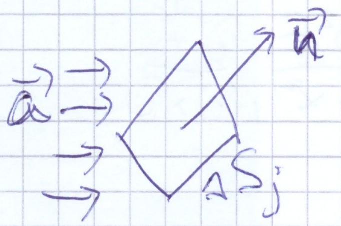
když $\vec{s} = (1,0,0)$, $\frac{\partial f}{\partial s} = (\text{grad } f)_x = \frac{\partial f}{\partial x}$

$\Rightarrow \text{grad } f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right)$

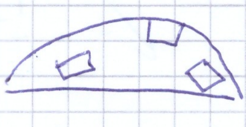
– symbol nabla (∇) $\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$

$\text{grad } f = \nabla f$

TOK VEKTOROVÉHO POLE PLOCHOU A DIVERGENCE



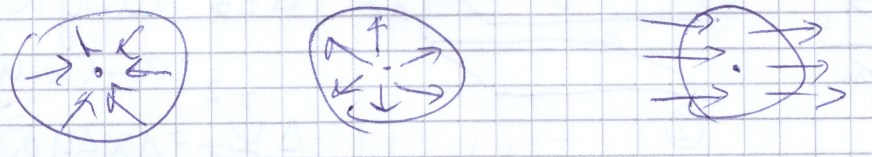
$\Phi_{S_j} = \Delta S_j \cdot \vec{a}\cdot\vec{n}$



obecná větší plocha limitně přes integrál

$$\Phi = \sum_j \Delta S_j \cdot \vec{a}(\vec{r})\cdot\vec{n}_j \xrightarrow{\Delta S\to 0} \boxed{\int \vec{a}(\vec{r})\cdot\vec{n}\, dS = \Phi}$$

budeme zkoumat tok „v okolí bodu" uvažováním uzavřené plochy, která obklopuje bod

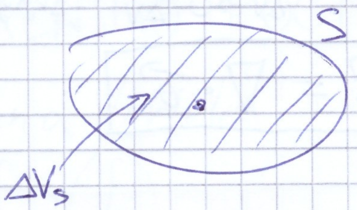


konvence: kladný tok = výtok ven, tedy limitně pro menší plochu blíže bodu

$$\Phi_S = \oint_S \vec{a}\cdot\vec{n}\, dS$$

když budeme tok normovat (ukazuje se, že dává smysl na objem):

$$\boxed{\frac{\Phi_S}{\Delta V_S} = \operatorname{div} \vec{a}}$$

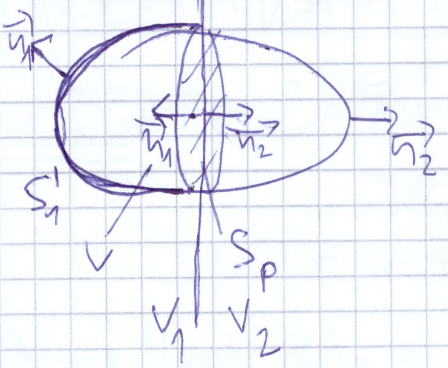


GAUSSOVA VĚTA INTEGRÁLNÍHO POČTU

$$\Phi_S = \sum_j \Phi_{S_j} = \sum \frac{\Phi_{S_j}}{\Delta V_j}\cdot \Delta V_j \xrightarrow[\Delta V_j \to 0]{\Delta S_j \to 0} \int \operatorname{div} \vec{a}\, dV$$

$$\Rightarrow \boxed{\oint_S \vec{a}\cdot\vec{n}\, dS = \int_{V(S)} \operatorname{div} \vec{a}\, dV}$$

proč to můžu rozseknout?

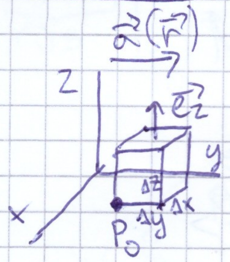


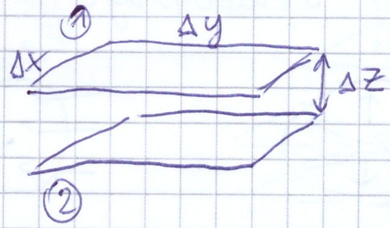
$$\Phi_1 = \oint_{S_1} \vec{a}\cdot\vec{n}\, dS$$

$$\Phi_2 = \oint_{S_2} \vec{a}\cdot\vec{n}\, dS$$

Φ_1, Φ_2 části původního povrchu a navíc 2x přepážka, ty se ale odečtou

DIVERGENCE V KARTÉZSKÝCH SOUŘADNICÍCH





$$\Delta \Phi_z^1 = \Delta x \Delta y\, \vec{a} \cdot \vec{e}_z = \Delta x \Delta y \cdot a_z(x,y,z+\Delta z)$$

$$\Delta \Phi_z^2 = \Delta x \Delta y\, \vec{a} \cdot (-\vec{e}_z) = \Delta x \Delta y \cdot (-a_z(x,y,z))$$

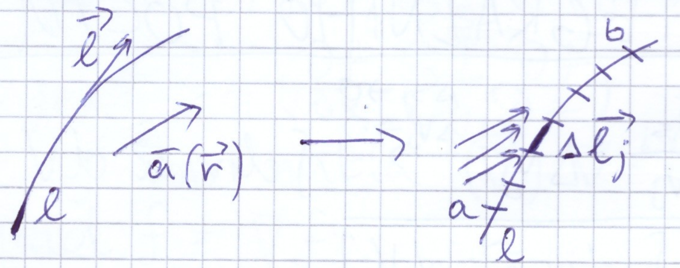
$$\Delta \Phi_z^{1+2} = \Delta x \Delta y \underbrace{\left[a_z(x,y,z+\Delta z) - a_z(x,y,z) \right]}_{\frac{\partial a_z}{\partial z} \cdot \Delta z} = \Delta x \Delta y \Delta z \frac{\partial a_z}{\partial z}$$

$$\Rightarrow \Delta \Phi_{x,y,z}^{1+2} = \Delta x \Delta y \Delta z \left[\frac{\partial a_x}{x} + \frac{\partial a_y}{y} + \frac{\partial a_z}{z} \right]$$

$$\Rightarrow \operatorname{div} \vec{a} = \lim_{\Delta V \to 0} \frac{\Phi}{\Delta V} = \lim_{\Delta x, \Delta y, \Delta z \to 0} \frac{\partial a_x}{\partial x} + \frac{\partial a_y}{y} + \frac{\partial a_z}{z}$$

$$\boxed{\operatorname{div} \vec{a} = \nabla \cdot \vec{a}}$$

ROTACE A KŘIVKOVÝ INTEGRÁL



když budu chtít primět vek. pole do křivky:

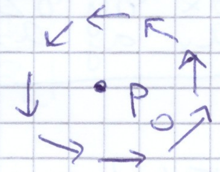
$\vec{a}(\vec{r}_j) \cdot \Delta \vec{l}_j$ (skal. souč. podobně jak u toku – záleží na směru)

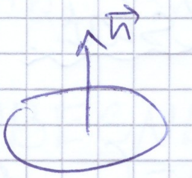
$$\rightarrow \sum \vec{a}(\vec{r}_j) \cdot \Delta \vec{l}_j \xrightarrow{\Delta l_j \to 0} \int_l \vec{a}(\vec{r}_j) \cdot d\vec{l}$$

speciálně po uzav. křivce $\oint$

CIRKULACE VEKTOROVÉHO POLE PO KŘIVCE

$$C_l(\vec{a}) = \oint_l \vec{a} \cdot d\vec{l}$$



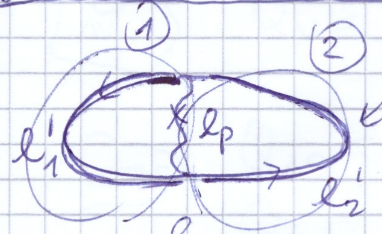


u rovinné křivky dobře určený směr
obdobně budu normovat (na obsah) a
vezmu v limitě

$$\boxed{\lim_{\Delta S_l \to 0} \frac{C_l(\vec{a})}{\Delta S_l} = \mathrm{rot}\,\vec{a} \cdot \vec{n}}$$

promítám do směru

STOKESOVA VĚTA INTEGRÁLNÍHO POČTU



obecně nerovinná křivka

$$C_{l_1}(\vec{a}) = \int_{l'_1} + \int_{l_p}$$

$$C_{l_2}(\vec{a}) = \int_{l'_2} + \int_{l_p}$$

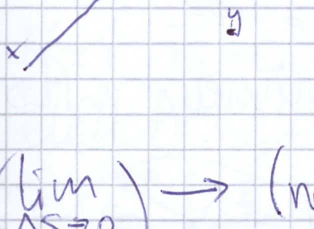
– v opač. směru vyruší se

$$\Rightarrow C_l(\vec{a}) = \sum_j C_{l_j}(\vec{a}) = \sum_j \frac{C_{l_j}(\vec{a})}{\Delta S_j} \cdot \Delta S_j \xrightarrow{\Delta S_j \to 0} \int_{S(l)} \mathrm{rot}\,\vec{a} \cdot \vec{n}\, dS.$$

$$\Rightarrow \boxed{\oint_l \vec{a}(\vec{l}) \cdot d\vec{l} = \int_{S(l)} \mathrm{rot}\,\vec{a} \cdot \vec{n}\, dS}$$

– platí $\forall$ plochy ohr. l !!!

ROTACE V KARTÉZSKÝCH SOUŘADNICÍCH



$$C = \Delta x \cdot a_x(x,y,z) + \Delta y \cdot a_y(x+\Delta x, y, z) -$$
$$- \Delta x \cdot a_x(x, y+\Delta y, z) - \Delta y \cdot a_y(x,y,z)$$
$$= -\frac{\partial a_x}{\partial y} \Delta y \cdot \Delta x + \frac{\partial a_y}{\partial x} \Delta y \Delta x$$

$$\left(\lim_{\substack{\Delta S \to 0 \\ \Delta x, \Delta y \to 0}}\right) \longrightarrow (\mathrm{rot}\,\vec{a})_z = \frac{\partial a_y}{\partial x} - \frac{\partial a_x}{\partial y}$$, obdobně pro x, y

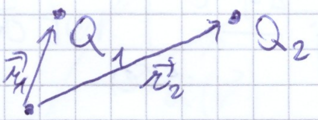
$$\Rightarrow \boxed{\nabla \times \vec{a} = \begin{vmatrix} e_x & e_y & e_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ a_x & a_y & a_z \end{vmatrix} = \mathrm{rot}\,\vec{a}}$$

3. Elektrostatika, východiska elstat.

Q, q ... elektrický náboj (skalár), $[Q]$ – C ... coulomb
- zákon zachování el. náboje
- invariance vůči pohybu (i v relat.)
- kladný nebo záporný, dá se sčítat
- kvantovaný $Q = p \cdot e$, e – elem. náboj, $p \in \mathbb{Z}$ (pro vol. č.)
(pozn. $p = \pm 1/3, \pm 2/3$ pro kvarky, $e = 1{,}602176634 \cdot 10^{-19}$ C)

COULOMBŮV ZÁKON

$\cdot Q$ bodový náboj



$$\boxed{\vec{F_{21}} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q_1 \cdot Q_2}{|\vec{r_2} - \vec{r_1}|^2} \cdot \frac{\vec{r_2} - \vec{r_1}}{|\vec{r_2} - \vec{r_1}|}}$$ COULOMBŮV ZÁKON

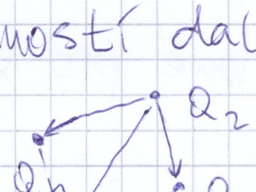
$Q_1 Q_2 > 0$ – $F > 0$ – náboje se odpuzují
$\quad\quad <$ $\quad\quad\quad <$ $\quad$ přitahují

ε_0 – permitivita vakua, $\varepsilon_0 \doteq 8{,}8541878128 \cdot 10^{-12}$ F/m

$\left[\frac{1}{4\pi\varepsilon_0}\right] = N \cdot \frac{1}{C^2} m^2 \Rightarrow [\varepsilon_0] = \frac{C^2}{N m^2}$

PRINCIP SUPERPOZICE

Silové působení mezi 2 náboji není ovlivněno přítomností dalších nábojů.



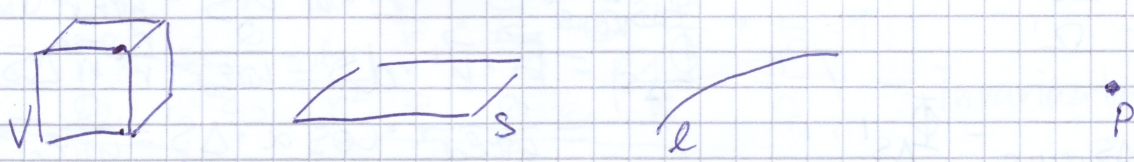
$\vec{F_{21}} = \frac{1}{4\pi\varepsilon_0} \frac{Q_1 Q_2}{|\vec{r_2} - \vec{r_1}|^2} \cdot \frac{\vec{r_2} - \vec{r_1}}{|\vec{r_2} - \vec{r_1}|}$, $\vec{F_{23}} = \dots$, $\vec{F_2} = \vec{F_{21}} + \vec{F_{23}}$

když vezmu na jednotkový náboj – intenzita el. pole

$$\vec{E}(\vec{r}_0) = \sum_{k=1}^{N} \frac{1}{4\pi\varepsilon_0} \frac{Q_k}{|\vec{r}_0 - \vec{r}_k|^2} \cdot \frac{\vec{r}_0 - \vec{r}_k}{|\vec{r}_0 - \vec{r}_k|}$$

$$\vec{F}_{Q_0} = Q_0 \cdot \vec{E}(\vec{r}_0), \quad [E] = \frac{N}{C}$$

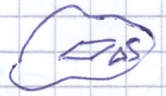
OBECNĚJŠÍ ROZLOŽENÍ NÁBOJE



objemový plošný délkový bodový

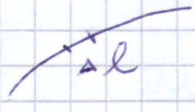
$Q_{\Delta V} = \varrho \Delta V$, kde ϱ je objemová hustota náboje

$Q_V = \int_V \varrho(\vec{r})\, dV$



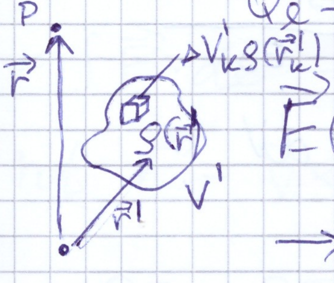
$Q_{\Delta S} = \sigma \Delta S$, kde σ je plošná hustota náboje

$Q_S = \int_S \sigma(\vec{r})\, dS$



$Q_{\Delta l} = \lambda \Delta l$, kde λ je délková hustota náboje

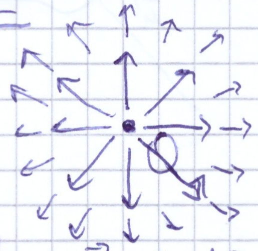
$Q_l = \int_l \lambda(\vec{r})\, dl$



$$\vec{E}(\vec{r}) = \sum_k \frac{1}{4\pi\varepsilon_0} \frac{\varrho(\vec{r}\,'_k)\Delta V'_k}{|\vec{r} - \vec{r}\,'_k|^2} \cdot \frac{\vec{r} - \vec{r}\,'_k}{|\vec{r} - \vec{r}\,'_k|} \xrightarrow{\Delta V'_k \to 0}$$

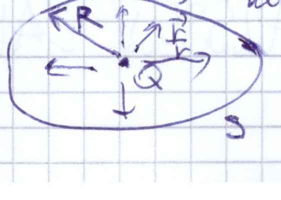
$$\to \boxed{\frac{1}{4\pi\varepsilon_0} \int_{V'} \frac{\varrho(\vec{r}\,')}{|\vec{r} - \vec{r}\,'|^2} \cdot \frac{\vec{r} - \vec{r}\,'}{|\vec{r} - \vec{r}\,'|}\, dV' = \vec{E}(\vec{r})}$$

VLASTNOSTI ELEKTRICKÉHO POLE



$N = 1, \ \vec{r}_1 = \vec{0} \qquad \vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q_1}{r^2} \cdot \frac{\vec{r}}{r}$

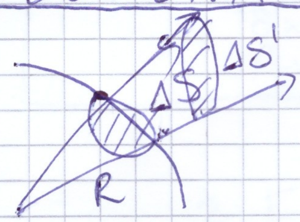
elektrické siločáry – křivky t.ž. $\vec{E}$ tečna



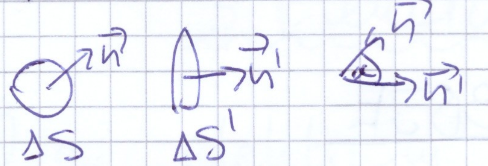
$$\Phi = \oint_S \vec{E} \cdot \vec{n}\, dS = \oint_S \frac{1}{4\pi\varepsilon_0} Q \frac{1}{r^2} \cdot \frac{\vec{r}}{r} \cdot \vec{n}\, dS =$$

$$= \frac{1}{4\pi\varepsilon_0} Q \oint_S \frac{1}{R^2} dS = \frac{4\pi R^2}{4\pi\varepsilon_0 R^2} Q = \boxed{\frac{Q}{\varepsilon_0} = \Phi}$$

LIBOVOLNÁ PLOCHA



$$d\Omega = \frac{\Delta S}{R^2}$$



$\Omega = \Omega'$

$$\Delta S = \Delta S' \cos\alpha$$

$$\Phi_{\Delta S_{KOULE}} = \cancel{[illegible]} \; \vec{E}\cdot\vec{n}\, dS = \frac{Q}{4\pi\varepsilon_0 r^2} \left(\frac{\vec{r}}{r}\cdot\vec{n}\right) \Delta S$$

$$\Phi_{\Delta S'} = \vec{E}\cdot\vec{n}'\, dS' = \frac{Q}{4\pi\varepsilon_0 r^2}\, \frac{\vec{r}}{r}\cdot\vec{n}'\, \Delta S' = \frac{Q}{4\pi\varepsilon_0 r^2}\cos\alpha \cdot \Delta S' = \frac{Q}{4\pi\varepsilon_0 r^2}\Delta S$$

$$\Rightarrow \Phi_{\Delta S_{KOULE}} = \Phi_{\Delta S'}$$

- nezávisí na tvaru plochy, pouze pokud je uvnitř, nebo venku

$$\Phi = \frac{Q}{\varepsilon_0}, \quad \vec{E} = \sum_{k=1}^{N} \vec{E}_k = \sum_k \frac{1}{4\pi\varepsilon_0}\frac{Q_k}{r_k^2}\frac{\vec{r}}{r}, \quad Q := Q_1 + \dots + Q_N$$

- superpozice i toků

Q-náboj uvnitř

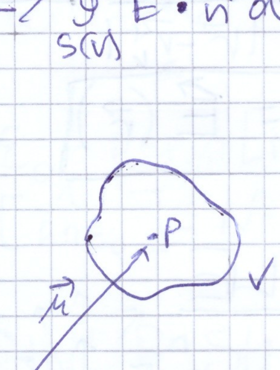
$$\Phi = \oint_S \vec{E}\cdot\vec{n}\, dS = \boxed{\frac{Q}{\varepsilon_0} = \Phi} \quad \text{GAUSSŮV ZÁKON}$$

$$Q = \int_{V(S)} \rho(\vec{r})\, dV$$

$$\Phi = \oint_S \vec{E}\cdot\vec{n}\, dS \; \cancel{[illegible]}, \quad \Phi = \frac{\int_{V(S)} \rho(\vec{r})\, dV}{\varepsilon_0}$$

$$\Rightarrow \oint_{S(V)} \vec{E}\cdot\vec{n}\, dS = \int_V \mathrm{div}\,\vec{E}\, dV = \frac{1}{\varepsilon_0}\int_V \rho(\vec{r})\, dV$$

$$\boxed{\int_V \left[\mathrm{div}\,\vec{E} - \frac{1}{\varepsilon_0}\rho\right] dV = 0}$$

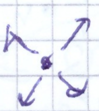


pro objem V platí, když budeme limitit k bodu

platí $V \to \Delta V$

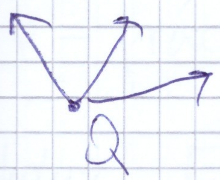
$$\left[\mathrm{div}\,\vec{E}(\vec{r}) - \frac{1}{\varepsilon_0}\rho(\vec{r})\right]\cdot V, \quad V \neq 0$$

$$\Rightarrow \boxed{\mathrm{div}\,\vec{E}(\vec{r}) = \frac{1}{\varepsilon_0}\rho(\vec{r})} \quad \text{GAUSSŮV ZÁKON}$$

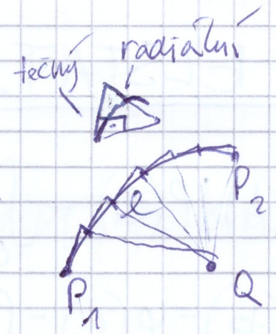


$\mathrm{div}\,\vec{E} > 0 \Leftrightarrow \rho > 0$ $\qquad$ $\mathrm{div}\,\vec{E} < 0 \Leftrightarrow \rho < 0$

ROTACE EL. POLE



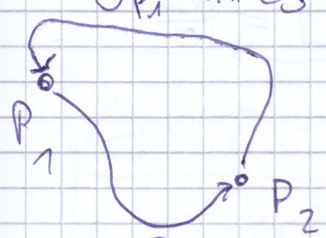
$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r^2} \cdot \frac{\vec{r}}{r}$$



$$\int_l \vec{E} \cdot d\vec{l}$$

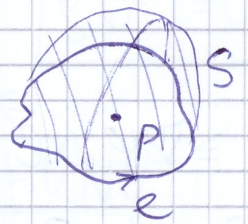
$$\int_{P_1}^{P_2} \vec{E} \cdot d\vec{l} \approx \sum_j \underbrace{\vec{E} \Delta \vec{l}_T}_{\frac{\vec{r}}{r} \cdot \vec{t} = 0} + \sum \underbrace{\vec{E} \Delta \vec{l}_R}_{\frac{\vec{r}}{r} \cdot \frac{\vec{r}}{r} = 1} = \frac{Q}{4\pi\varepsilon_0} \sum_j \frac{1}{r^2} \Delta r_j \rightarrow$$

$$\rightarrow \int_{P_1}^{P_2} \frac{Q}{4\pi\varepsilon_0} \frac{dr}{r^2} = -\frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_{P_2}} - \frac{1}{r_{P_1}}\right)$$ – invariantní vůči cestě



$$\boxed{\oint_l \vec{E} \cdot d\vec{l} = 0}$$, když nyní použiju Stok. v.

$$\oint_l \vec{E} \cdot d\vec{l} = \int_{S(l)} \mathrm{rot}\, \vec{E} \cdot \vec{n}\, dS = 0$$



když budu limitit l, S k bodu P

$$\boxed{\mathrm{rot}\, \vec{E} = 0}$$

$\vec{F}_Q = Q\vec{E}$, pro jednot. náb. $\int_{P_1}^{P_2} \vec{E} \cdot d\vec{l} \rightsquigarrow Q \int \vec{E}\, d\vec{l} =$

$$= \int \vec{F} \cdot d\vec{l} = W = 0$$

$$\mathrm{rot}\, \vec{E} = \nabla \times \vec{E} = \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = 0$$

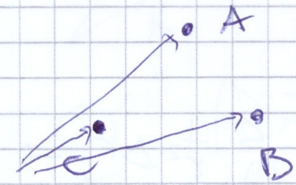
$$\Rightarrow \frac{\partial E_x}{\partial y} = \frac{\partial E_y}{\partial x}$$

$$\frac{\partial E_z}{\partial y} = \frac{\partial E_y}{\partial z}$$

$$\frac{\partial E_z}{\partial x} = \frac{\partial E_x}{\partial z}$$

POTENCIÁL ELSTAT. POLE

$\varphi = \varphi(\vec{r})$ $\quad \varphi(A) = \varphi(C) - \int_C^A \vec{E} \cdot d\vec{l}$

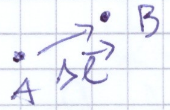


$\varphi(B) = \varphi(C) - \int_C^B \vec{E} \cdot d\vec{l}$

$\varphi(B) - \varphi(A) = \int_C^A \vec{E} \cdot d\vec{l} - \int_C^B \vec{E} \cdot d\vec{l} =$

$= -\int_A^B \vec{E} \cdot d\vec{l}$ – nejde o abs. h. φ, ale o změnu φ od bodu k bodu, $[\varphi] = \frac{Nm}{C}$

POTENCIÁL V KART. SOUŘ.



$\Delta\varphi = -\vec{E} \cdot d\vec{l}$

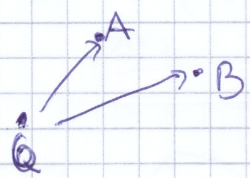
$\Delta\varphi = \frac{\partial \varphi}{\partial l} \cdot \Delta l \overset{\text{der. ve sm. } \Delta l}{=} \text{grad}\, \varphi \cdot \frac{\Delta \vec{l}}{\Delta l} \cdot \Delta l = \nabla\varphi \Delta\vec{l}$

$-\vec{E} \cdot d\vec{l} = \nabla\varphi \Delta\vec{l} \xrightarrow{\Delta\vec{l} \to 0} [\nabla\varphi - \vec{E}] d\vec{l} = 0$

$\Rightarrow \boxed{\vec{E} = -\nabla\varphi}$

formálně: potenciál je skalární fce v prostoru t. ž.

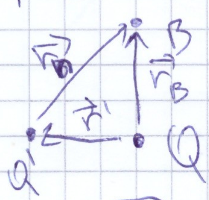
$\nabla\varphi = -\vec{E}$, $\text{rot}\, \vec{E} = 0$ – konz. pole



$\int_A^B \vec{E} \cdot d\vec{l} = \frac{-Q}{4\pi\varepsilon_0} \left(\frac{1}{r_B} - \frac{1}{r_A} \right)$ – rozdíl potenciálů

$\vec{E} = \frac{Q}{4\pi\varepsilon_0} \cdot \frac{1}{r^2} \cdot \frac{\vec{r}}{r}$

když $r_A \to \infty$, $\varphi(B) - \varphi(A) = \frac{-Q}{4\pi\varepsilon_0} \cdot \frac{1}{r_B}$ – vztahujeme potenciál k nekonečnu, asi nejrozumnější volba



$\vec{r}_n = \vec{r}_B - \vec{r}_n$ – posun Q, použiji pro objem. pole

$|\vec{r}_n| = |\vec{r}_B - \vec{r}_N|$

$\vec{E} = \sum_j \vec{E}_j$

$\Rightarrow \varphi = \sum_j \varphi_j$

$$\Rightarrow \varphi(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \int_V \frac{\rho(\vec{r}\,')\,dV'}{|\vec{r}-\vec{r}\,'|}$$

φ – práce na 1 C, ale pozor na znaménko

když $\vec{E} = -\nabla\varphi$, $\nabla\cdot\vec{E} = \frac{\rho}{\varepsilon_0}$

$$\nabla\cdot\vec{E} = -\nabla\cdot\nabla\varphi$$

$$\nabla\cdot\nabla\varphi = -\frac{\rho}{\varepsilon_0} = \nabla\cdot\left(\frac{\partial\varphi}{\partial x}, \frac{\partial\varphi}{\partial y}, \frac{\partial\varphi}{\partial z}\right) = \underbrace{\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)}\varphi$$

$\boxed{\Delta\varphi(\vec{r}) = -\frac{\rho(\vec{r})}{\varepsilon_0}}$ POISSONOVA RCE

Δ LAPLACEŮV
∇^2 OPERÁTOR

$\boxed{\rho(\vec{r}) = 0 : \Delta\varphi(\vec{r}) = 0}$ LAPLACEOVA RCE

SHRNUTÍ

$$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q_1 Q_2}{|\vec{r}_1-\vec{r}_2|^2}\cdot\frac{\vec{r}_1-\vec{r}_2}{|\vec{r}_1-\vec{r}_2|}$$ COULOMB

$\nabla\cdot\vec{E} = \frac{\rho}{\varepsilon_0}$

GAUSS

$\oint_S \vec{E}\cdot\vec{n}\,dS$

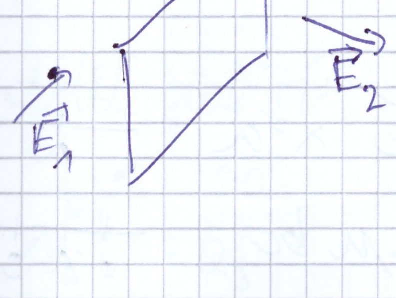
$\nabla\times\vec{E} = 0$

ROTACE

$\oint_l \vec{E}\cdot d\vec{l} = 0$

$\vec{E} = -\nabla\varphi$ POTENCIÁL

ELSTAT. NA ROZHRANÍ

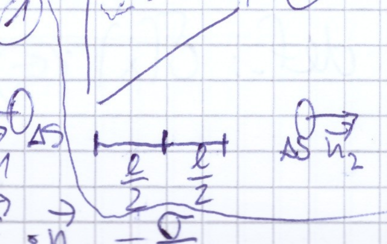


normálové složky:

$$\vec{E}_1\cdot\vec{n}_1\Delta S + \vec{E}_2\cdot\vec{n}_2\Delta S + \int_{PLÁŠŤ_1} + \int_{PLÁŠŤ_2} = \frac{\sigma\Delta S}{\varepsilon_0}$$

$$\xrightarrow{l\to 0} [(\vec{E}_1\cdot\vec{n}_1) + (\vec{E}_2\cdot\vec{n}_2)]\Delta S = \frac{\sigma\Delta S}{\varepsilon_0} \Rightarrow \vec{E}_1\cdot\vec{n}_1 + \vec{E}_2\cdot\vec{n}_2 = \frac{\sigma}{\varepsilon_0}$$

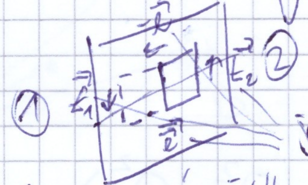
„Gaussova plechovka"



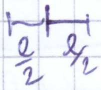
ale $\vec{n}_1 = -\vec{n}_2 \Rightarrow$ $\boxed{\begin{aligned}(\vec{E}_2 - \vec{E}_1)\cdot\vec{n}_2 &= \frac{\sigma}{\varepsilon_0}\\ E_{2n} - E_{1n} &= \frac{\sigma}{\varepsilon_0}\end{aligned}}$

tečné složky:

$$\oint_{\text{RÁMEČEK}} \vec{E}\cdot d\vec{l} = 0$$



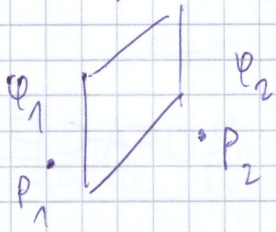
„Gaussův rámeček"



$$\vec{E}_1\cdot\vec{t}_1\cdot d + \cancel{\vec{E}_1\cdot\vec{l}\cdot\frac{l}{2}} + \cancel{\vec{E}_2\cdot\vec{l}\cdot\frac{l}{2}} +$$
$$+\vec{E}_2\cdot\vec{t}_2\cdot d - \cancel{\vec{E}_2\cdot\vec{l}\cdot\frac{l}{2}} - \cancel{\vec{E}_1\cdot\vec{l}\cdot\frac{l}{2}} = 0$$
$$\xrightarrow{l\to 0} \vec{E}_1\cdot\vec{t}_1\cdot d + \vec{E}_2\cdot\vec{t}_2\cdot d = 0$$
$$\vec{E}_1\cdot\vec{t}_1 + \vec{E}_2\cdot\vec{t}_2 = 0$$

ale $\vec{t}_1 = -\vec{t}_2 \Rightarrow$ $\boxed{\begin{aligned}(\vec{E}_2 - \vec{E}_1)\cdot\vec{t}_2 &= 0\\ E_{2t} - E_{1t} &= 0\end{aligned}}$

a co potenciál?



$\boxed{\Delta\varphi = 0}$ $\quad \Delta\varphi = -\vec{E}\cdot d\vec{l}$, $d\vec{l} \to 0$, $P_1 \to P_2$

*DIRACOVA δ-FCE

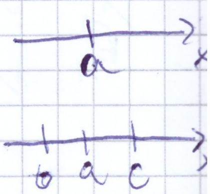
$\delta: f(x) \mapsto f(a)$, $f: \mathbb{R}\to\mathbb{R}$

t.z. $f(a) = \int_{-\infty}^{\infty} f(x)\,\delta(x-a)\,dx$

def.: $\delta(x)$: $\delta(x-a) = 0$, kromě $x = a$

$\int_b^c \delta(x-a)\,dx = 1$, pokud $a \in bc$

$= 0$ jinde



– „limita" Gaussovy distribuce pro menší šířku/rozptyl

použití: $\vec{E} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r^2} \cdot \frac{\vec{r}}{r}$ $\quad \overset{?}{:} \operatorname{div}\left(\frac{\vec{r}}{r^3}\right) =$

$\odot_Q$ pro $\vec{r} = \vec{0}$ $\qquad = 3 \cdot \frac{1}{r^3} - 3 \cdot \frac{r^2}{r^5} = 0$

vyřeším udeláním nenulové koule kolem náboje, div venku je nula, vevnitř limitím pro $R \to 0$, ale pořád počítám i objemu

$$\int_V \nabla \cdot \left(\frac{\vec{r}}{r^3}\right) dV \overset{G.V.}{\underset{i.p.}{=}} \oint_{S(V)} \frac{1}{r^2} \cdot \underbrace{\frac{\vec{r}}{r} \cdot \vec{n}}_{1} \cdot dS = \frac{1}{R^2} \cdot \overbrace{\oint_{S(V)} dS}^{4\pi R^2} = 4\pi$$

– nezávisí na R

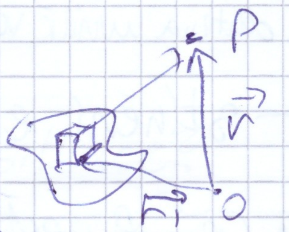
$$\Rightarrow \operatorname{div}\left(\frac{\vec{r}}{r^3}\right) = \begin{cases} 0 & r \neq 0 \\ 4\pi & r \to 0 \end{cases}$$

$$\delta^3(\vec{r} - \vec{a}) = \delta(x - a_x)\,\delta(y - a_y)\,\delta(z - a_z) \quad , a = 0$$

$$\nabla\left(\frac{\vec{r}}{r^3}\right) = 4\pi \cdot \delta^3(\vec{r})$$, když posunu $\overset{\bullet}{Q} \xleftarrow{\vec{r}'} O$

$$\nabla \cdot \frac{\vec{r} - \vec{r}'}{|\vec{r} - \vec{r}'|^3} = 4\pi\, \delta^3(\vec{r} - \vec{r}')$$

když objemový náboj



$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \int_{V'} \frac{\vec{r} - \vec{r}'}{|\vec{r} - \vec{r}'|^3}\, \varrho(\vec{r}')\, dV'$$

čárk.

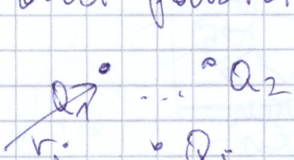
$$\nabla_{\vec{r}} \cdot \vec{E} = \frac{1}{4\pi\varepsilon_0} \int_{V'} \varrho(\vec{r}')\, \nabla \cdot \left(\frac{\vec{r} - \vec{r}'}{|\vec{r} - \vec{r}'|^3}\right) dV' =$$

nečárk.

$$= \frac{1}{4\pi\varepsilon_0} \int_{V'} 4\pi\, \delta(\vec{r} - \vec{r}')\, \varrho(\vec{r}')\, dV' = \underline{\frac{1}{\varepsilon_0} \varrho(\vec{r})}$$

hle, Gauss

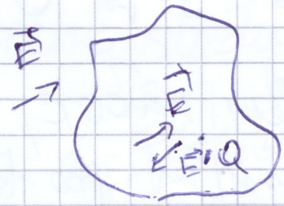
další použití:



$$\sum_j Q_j \cdot \delta(\vec{r} - \vec{r}_j) = \varrho(\vec{r})$$

VODIČE V ELSTAT.

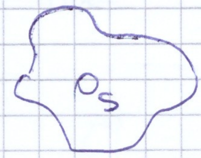
kovy mají delokalizované e^- ≈ vodič pro elstat.

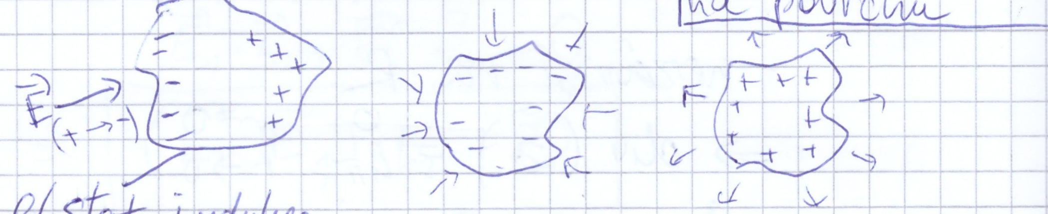


$\vec{F} = Q\vec{E}$, chceme stac. případ, takže

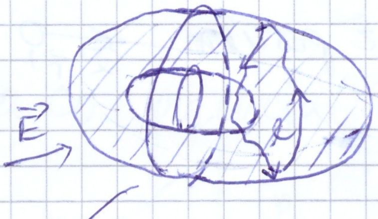
$\vec{E}_{UVNITŘ} = \vec{0}$ (náboje se rozmístí, aby vykompenzovaly pole)

$\oint_S \vec{E} \cdot \vec{n}\, dS = \frac{Q_{UVNITŘ}}{\varepsilon_0} \Rightarrow Q_{UVNITŘ} = 0$ – náboj se rozmístí na povrchu





elstat. indukce



dutina uvnitř vodiče

$\oint_l \vec{E} \cdot d\vec{l} = 0$

$\int_{VODIČ} \vec{E} \cdot d\vec{l} + \int_{DUTINA} \vec{E} \cdot d\vec{l} = 0$ pro lib l

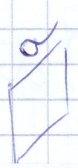
$\Rightarrow \vec{E}_{DUTINA} = \vec{0}$ (pokud $Q_{DUTINA} = 0$)

– stínění el. → Faradayova klec

a co φ potenciál? $\Delta\varphi = -\vec{E} \cdot d\vec{l}$, ale $\vec{E} = \vec{0}$

$\Rightarrow \Delta\varphi = 0$ – ve vodiči konst. potenciál (včetně povrchu)

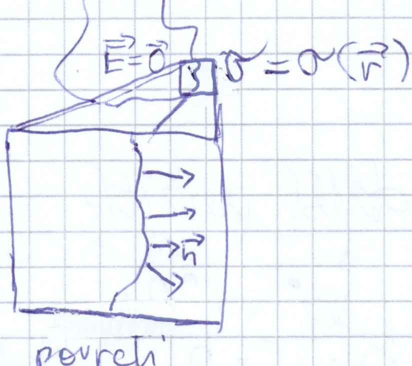
viz dříve



$\Delta E_n = \frac{\sigma}{\varepsilon_0}$, pro vodič

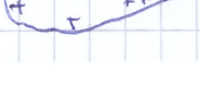
$\Delta E_t = 0$

$\Rightarrow \boxed{\vec{E} = \frac{\sigma}{\varepsilon_0}\vec{n}}$ COULOMBOVA VĚTA

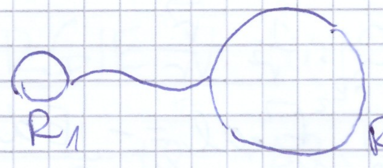


povrch

důsledek: hromadění náboje na hrot



Př.:



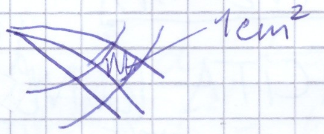
$R_2 > R_1$

$\varphi_1 = \frac{1}{4\pi\varepsilon_0}\frac{Q_1}{R_1}$ $\quad \varphi_2 = \frac{1}{4\pi\varepsilon_0}\frac{Q_2}{R_2}$

– spojím koule $\varphi_1 = \varphi_2 \Rightarrow \frac{Q_1}{R_1} = \frac{Q_2}{R_2}$; $Q_j = 4\pi R_j^2 \sigma_j$

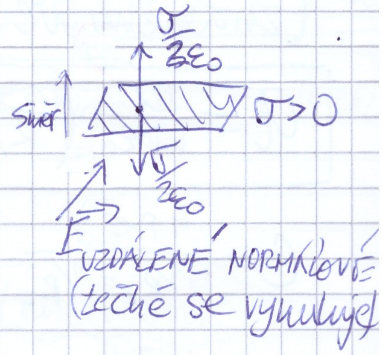
$\Rightarrow R_1\sigma_1 = R_2\sigma_2 \Rightarrow \sigma_1 = \frac{R_2}{R_1}\sigma_2$ – $\sigma_1 > \sigma_2$

SÍLA/TLAK NA POVRCH VODIČE



$\Delta Q = \sigma \Delta S$

$\Delta \vec{f} = \vec{E} \cdot \Delta Q$



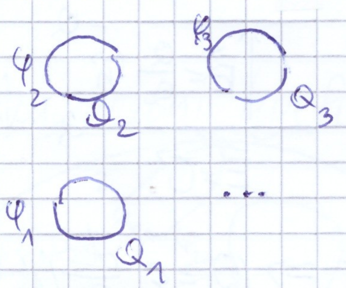
$\rightarrow E_{\text{VZDÁLENÉ NORMÁLOVÉ}} - \frac{\sigma}{2\varepsilon_0} = 0$

$E_{V_n} = \frac{\sigma}{2\varepsilon_0}$

celkové $\vec{E}_{V_n} + \vec{E}_{\sigma_2} = \frac{\sigma}{\varepsilon_0}$ – G. věta platí $\Rightarrow \Delta\vec{f} = \frac{\sigma}{2\varepsilon_0}\sigma\Delta S\vec{n}$

tlak: $\vec{p} = \frac{\Delta \vec{f}}{\Delta S} = \frac{\sigma^2}{2\varepsilon_0}\vec{n}$

ZÁKLADNÍ ÚLOHA ELSTAT.

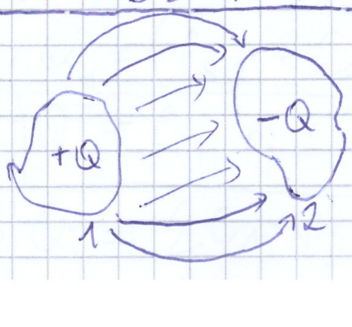


Ustálí se lib. výchozí situace?
Je toto ustálení jednoznačné?

Ano a ano.

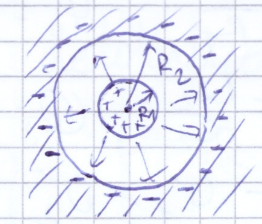
pro popis použijeme Laplaceovu rci $\Delta\varphi = 0$ – do PC

KONDENZÁTOR



2 vodivá tělesa, která mají stejný, ale opačný náboj a siločáry z jednoho končí v tom druhém.

prakticky: koule



$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \cdot Q \cdot \frac{1}{r^2} \cdot \frac{\vec{r}}{r}$$

$$\varphi_2 - \varphi_1 = -\int_{R_1}^{R_2} \vec{E} \cdot d\vec{l} = -\int_{R_1}^{R_2} E\, dr =$$

$$= -\frac{Q}{4\pi\varepsilon_0} \int_{R_1}^{R_2} \frac{dr}{r^2} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{R_2} - \frac{1}{R_1}\right)$$

místo potenciálu se zavede ELEKTRICKÉ NAPĚTÍ (znaménkově stejné jako práce el. pole) $\boxed{U = -(\varphi_2 - \varphi_1)}$

$$\boxed{U = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)} \qquad \boxed{U = \frac{1}{C} Q} \text{ KAPACITA KOND.}$$

$$C = \frac{4\pi\varepsilon_0}{1/R_1 - 1/R_2} \quad \text{pro kulový kond.}$$

$$[U] = V = \frac{Nm}{C} \quad \text{(volt)}$$

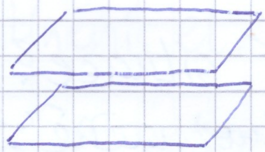
$$\boxed{Q = CU} \quad \Rightarrow [E] = \frac{V}{m}$$

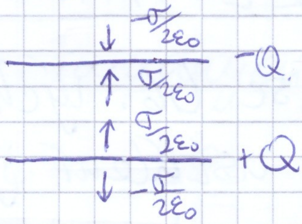
$$[C] = F \text{ – farad}$$

Př.: kapacita „osamocené koule" – $R_2 \to \infty$

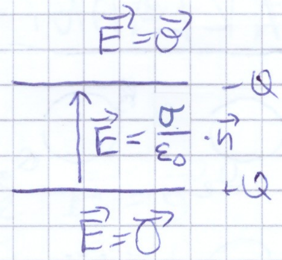
$C = 4\pi\varepsilon_0 R_1$ (jednotka metr?)

deskový kondenzátor:





směr →



$$\sigma = \frac{Q}{S} \qquad U = E \cdot d = \frac{\sigma d}{\varepsilon_0} \Rightarrow \frac{U}{d} = E = \frac{Q}{S\varepsilon_0} \Rightarrow Q = \varepsilon_0 \frac{S}{d} U$$

$$(Q = C \cdot U)$$

$$\Rightarrow \boxed{C = \varepsilon_0 \frac{S}{d}}$$ – znaménko nás moc nezajímá, pouze konvence

ENERGIE V KOND.

$+Q \quad +\Delta Q$

$-Q \quad -\Delta Q$

) vykonám práci – dávám do kond. energii

$\Delta W = \Delta Q \cdot U$, $Q = CU$, $\Delta Q = C \Delta U$

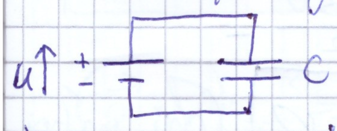
$$\Delta W = \Delta Q \cdot \frac{Q}{C} \xrightarrow{\int_{ZAČ.}^{KON.}} W = \int_0^{Q_K} \frac{Q}{C}\, dQ = \frac{1}{C} \cdot \left[\frac{1}{2} Q^2\right]_0^{Q_K} \boxed{W = \frac{Q_K^2}{2C} = \frac{1}{2} C U^2}$$

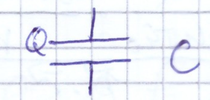
$$U_K = E \cdot d, \quad C = \varepsilon_0 \frac{S}{d} \Rightarrow W = \frac{1}{2} \varepsilon_0 \frac{S}{d} d^2 E^2 = \frac{1}{2} \varepsilon_0 V_{KOND} E^2 \text{ – E intenz.}$$

$\Rightarrow \boxed{w_e = \frac{1}{2} \varepsilon_0 E^2}$ – „hustota" energie el. pole

změna kapacity:

? ΔW, když ? ΔC

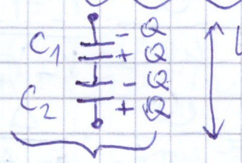




U konst. Q konst.

$$\Delta W = \frac{1}{2} U^2 \cdot \Delta C \qquad \Delta W = \frac{1}{2} Q^2 \cdot \Delta\left(\frac{1}{C}\right) = \frac{1}{2} Q^2 \frac{1}{C^2} \Delta C =$$
$$= -\frac{1}{2} U^2 \Delta C$$

řazení kondenzátorů:



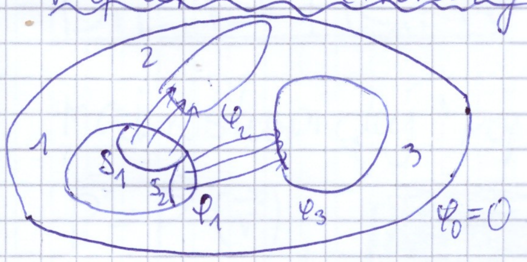
$Q_j = C_j \cdot U_j$

$$U = \sum_j \frac{Q_j}{C_j} = Q \boxed{\sum \frac{1}{C_j} = C}$$

$Q_j = C_j \cdot U_j$

$$Q = \sum_j U_j C_j = U \boxed{\sum C_j = C}$$

*kapacitní koeficienty:



$$U_{21} = -(\varphi_2 - \varphi_1)$$

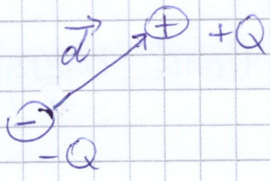
$$\left.\begin{aligned} Q_{S_1} &= C_{S_1 2} \cdot U_{21} \\ Q_{S_2} &= C_{S_2 3} \cdot U_{31} \\ &\dots \end{aligned}\right\} \begin{aligned} Q_1 &= C_{S_1 2} U_{21} + C_{S_2 3} U_{31} + \dots \\ Q_1 &= -C_{S_1 2}(\varphi_2 - \varphi_1) - C_{S_2 3} \cdot (\varphi_3 - \varphi_1) + \dots \end{aligned}$$

$$\Rightarrow Q_1 = (C_{S_1 2} + C_{S_2 3} + \dots)\varphi_1 - C_{S_1 2}\varphi_2 - C_{S_2 3}\varphi_3 - \dots$$

$\boxed{Q_1 = C_{11}\varphi_1 + C_{12}\varphi_2 + C_{13}\varphi_3 + \dots}$, C_{ij} – kapacitní koeficienty

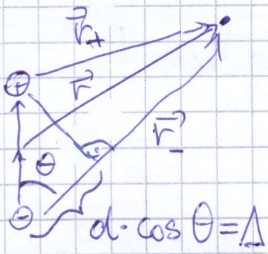
$\Rightarrow Q_i = \sum_j C_{ij} \varphi_j$ – SLR $\rightarrow \varphi_j = \sum_i B_{ij} Q_i$, B_{ij} – influenční koef.

ELEKTRICKÝ DIPÓL



dipólový moment: $\boxed{\vec{p} = Q \cdot \vec{d}}$

EL. POLE DIPÓLU



$\vec{p} = \vec{p}(\vec{r})$

$$\varphi = \frac{1}{4\pi\varepsilon_0} \cdot \frac{+Q}{r_+} + \frac{1}{4\pi\varepsilon_0} \cdot \frac{-Q}{r_-} = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{r_+} - \frac{Q}{r_-}\right)$$

$d \cdot \cos\theta = \Delta$ elementární (bodový) dipól: $\vec{p}$ konečně velké

$d \to 0$ $(d \ll r_+, r_-, r)$

$$\left.\begin{aligned} r_+ &\approx r - \frac{\Delta}{2} \\ r_- &\approx r + \frac{\Delta}{2} \end{aligned}\right\} \Rightarrow \varphi = \frac{Q}{4\pi\varepsilon_0} \frac{r_- - r_+}{r_- r_+} \approx$$

$$\approx \frac{Q}{4\pi\varepsilon_0} \cdot \frac{\Delta}{(r - \frac{\Delta}{2})(r + \frac{\Delta}{2})} \approx \frac{Q}{4\pi\varepsilon_0} \cdot \frac{d \cdot \cos\theta}{r^2} =$$

$\cos\theta = \frac{\vec{d}}{d} \cdot \frac{\vec{r}}{r}$

$$= \frac{1}{4\pi\varepsilon_0} \cdot Q \cdot \frac{\vec{d} \cdot \frac{\vec{r}}{r}}{r^2} = \boxed{\frac{1}{4\pi\varepsilon_0} \cdot \frac{\vec{p} \cdot \vec{r}}{r^3} = \varphi}$$

(pro náboj v počátku)

zajímavý

$$\vec{E} = -\nabla\varphi$$

$$(\nabla\varphi)_x = \frac{\partial}{\partial x} \cdot \frac{1}{4\pi\varepsilon_0} \cdot \frac{p_x x + p_y y + p_z z}{(x^2+y^2+z^2)^{3/2}} = \frac{1}{4\pi\varepsilon_0} \cdot \left(\frac{p_x}{(x^2+y^2+z^2)^{3/2}} + \frac{p_x x + p_y y + p_z z}{(x^2+y^2+z^2)^{5/2}} \cdot\right.$$

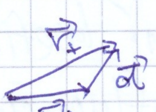
$$\left. \cdot -\frac{3}{2} \cdot 2x\right) = \frac{1}{4\pi\varepsilon_0}\left(\frac{p_x}{r^3} - 3\frac{\vec{p} \cdot \vec{r}}{r^5} x\right)$$

$$\vec{E} = -\frac{1}{4\pi\varepsilon_0}\left(\frac{\vec{p}}{r^3} - 3\frac{(\vec{p} \cdot \vec{r})\vec{r}}{r^5}\right)$$

$$\boxed{\vec{E} = \frac{1}{4\pi\varepsilon_0}\left(3\frac{(\vec{p} \cdot \vec{r})\vec{r}}{r^5} - \frac{\vec{p}}{r^3}\right)}$$

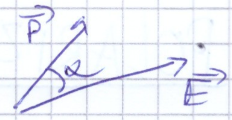
EL. DIPÓL VE VNĚJŠÍM POLI

energie: $W = \varphi(\vec{r}_+) \cdot Q - \varphi(\vec{r}_-) Q = Q \underbrace{(\varphi(\vec{r}_+) - \varphi(\vec{r}_-))}_{\underbrace{\frac{\vec{d}}{d} \cdot \nabla\varphi\, d}_{\text{der. ve sm. } \vec{d}}}$

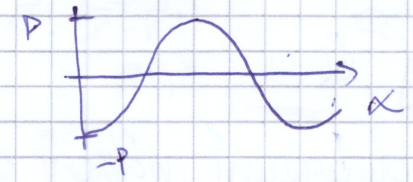


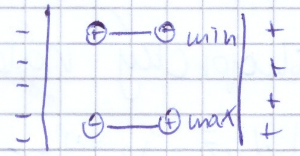
$\vec{r}_- + \vec{d} = \vec{r}_+$, $d \ll r_+, r_-$

$$W = Q\vec{d}\cdot\nabla\varphi = \vec{p}\cdot\nabla\varphi = -\vec{p}\cdot\vec{E} = -p\cdot E\cdot\cos\alpha$$

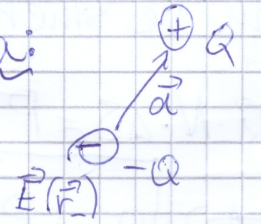


$$\boxed{W = -\vec{p}\cdot\vec{E}}$$



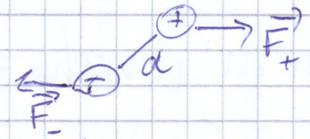


síla:



$$\vec{F} = Q\vec{E}(\vec{r}_+) - Q\vec{E}(\vec{r}_-)$$

pro homo. pole: $\vec{E}(\vec{r}) = \vec{E}_0$ $\quad \vec{F} = \vec{0}$, ale



$$M = dQ\,E_0 \sin\alpha$$

$$\boxed{\vec{M} = \vec{p}\times\vec{E}_0}$$

$$F_x = Q\,\underbrace{\left[E_x(\vec{r}_+) - E_x(\vec{r}_-)\right]}_{\frac{\vec{d}}{d}\cdot\nabla E_x\cdot d} = (\vec{p}\cdot\nabla)E_x$$

POZOR

$$\boxed{\vec{F} = (\vec{p}\cdot\nabla)\vec{E}} \qquad (\vec{p}\cdot\nabla) = \left(p_x\frac{\partial}{\partial x} + p_y\frac{\partial}{\partial y} + p_z\frac{\partial}{\partial z}\right)$$

$$(\vec{p}\cdot\nabla)\cdot\vec{E} = p_x\frac{\partial}{\partial x}E_x + p_y\frac{\partial}{\partial y}E_y + p_z\frac{\partial}{\partial z}E_z$$, když $\operatorname{rot}\vec{E} = \vec{0}$

$$\frac{\partial}{\partial x}(p_xE_x + p_yE_y + p_zE_z) = \frac{\partial}{\partial x}(\vec{p}\cdot\vec{E}) \Rightarrow$$

$$\Rightarrow \boxed{\vec{F} = \nabla(\vec{p}\cdot\vec{E})}$$

pro N objemovou hustotu dipólů:

$$\vec{f} = N\vec{F} = (N\vec{p}\cdot\nabla)\vec{E} = (\vec{P}\cdot\nabla)\vec{E} =$$

viz DIELEKTRIKA $\quad$ viz VLASTNOSTI DIEL.

$$= \varepsilon_0\chi(\vec{E}\cdot\nabla)\vec{E} \xrightarrow[\nabla\times\vec{E}=\vec{0}]{\text{roz. po složkách}} \frac{1}{2}\varepsilon_0\chi\nabla E^2$$

pro řídká pružná diel.

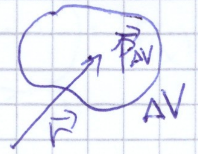
DIELEKTRIKA

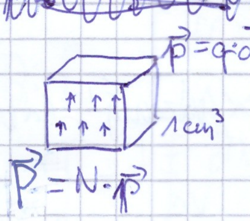
makroskopický popis vektorem elektrické polarizace:

$\vec{P} = \vec{P}(\vec{r})$, ~~[illegible]~~

$$\boxed{\vec{P}(\vec{r}) = \lim_{\Delta V\to 0}\frac{\vec{P}_{\Delta V}}{\Delta V}}$$ (v bodu daným $\vec{r}$)

(Úmluva: q – mikro náboj, Q – makro náboj)



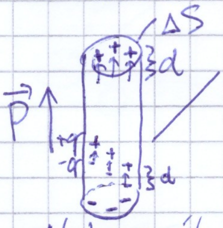


$$\vec{P}_{\Delta V} = \int_V \vec{P}(\vec{r})\,dV$$

Formálně pomocí δ-fce: $\boxed{\vec{P}(\vec{r}) = \sum_j \vec{p}_j(\vec{r}_j)\cdot\delta(\vec{r}-\vec{r}_j)}$

POLARIZOVANÉ DIELEKTRIKUM $\vec{P} \neq \vec{0}$

? makroskopický náboj Q



když je to homogenní, náboj uvnitř se vykompenzuje, zůstane náboj na povrchu ve směru $\vec{P}$

N-konc. náb.

$$\Delta Q = q \cdot N \cdot \Delta V = q \cdot N \cdot \Delta S \cdot d = p \cdot N \cdot \Delta S = P \cdot \Delta S$$

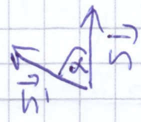
ΔQ jen na povrchu $\Rightarrow \Delta Q = \sigma_{P\Delta S} \cdot \Delta S \Rightarrow \boxed{\sigma_{P\Delta S} = P}$

$\Delta S'$ – nakloněná

ΔS – kolmá

$\vec{n} = \frac{\vec{P}}{P}$

$$\Rightarrow \sigma_{P\Delta S'} = \frac{\Delta Q}{\Delta S'} = \frac{\overbrace{\Delta Q}^{P}}{\Delta S} \vec{n} \cdot \vec{n}' = P(\vec{n} \cdot \vec{n}') = \boxed{\vec{P} \cdot \vec{n}' = \sigma_{P\Delta S'}}$$



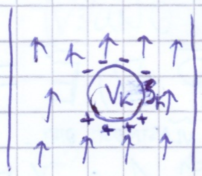
$\Delta S' \cos\alpha = \Delta S$

$\Delta S' \, \vec{n} \cdot \vec{n}' = \Delta S$

závisí na $\vec{r}$

$$[\sigma_p] = \frac{C}{m^2} \Rightarrow [P] = \frac{C}{m^2} \Leftarrow [p] = C \cdot m$$

OBJEMOVÝ NÁBOJ



$\uparrow \vec{P}$

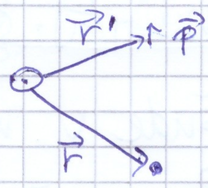
$$Q_{S_k} = \oint_{S_k(V_k)} \sigma_p(\vec{r})\, dS = \oint_{S_k(V_k)} \vec{P} \cdot \vec{n}\, dS \overset{G.V.}{=} \int_{V_k} \nabla \cdot \vec{P}\, dV$$

uvnitř není náboj $\left\{ Q_{V_k} = -Q_{S_k} \Rightarrow \int_{V_k} \rho_{V_k}(\vec{r})\, dV = -\int_{V_k} \nabla \cdot \vec{P}\, dV \right.$

$$\xRightarrow[\text{v bodu}]{V \to 0} \boxed{\rho_{V_k}(\vec{r}) = -\nabla \cdot \vec{P}(\vec{r})}$$

hustota „fiktivního" náboje uvnitř mezery vyvolaného polarizací – tzv. „vázaný náboj"

Formálně přes potenciál:



$$\varphi(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \cdot \frac{\vec{p}\cdot(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}$$

$$\vec{p} = \vec{P}\Delta V'$$

$$\Delta\varphi(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \cdot \frac{\vec{P}(\vec{r}')\,\Delta V' \cdot (\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}$$

$$\varphi(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \int\limits_{\substack{V' \\ \text{přes } \vec{r}'}} \frac{\vec{P}(\vec{r}')\cdot(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}\, dV' \qquad dx'dy'dz'$$

protože $\dfrac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3} = \nabla' \dfrac{1}{|\vec{r}-\vec{r}'|}$ (viz cvičení)

$$\varphi(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \int\limits_{\substack{V' \\ \text{přes } \vec{r}'}} \vec{P}(\vec{r}')\cdot\nabla' \frac{1}{|\vec{r}-\vec{r}'|}\, dV'$$

protože $\nabla(f\cdot\vec{a}) = f(\nabla\cdot\vec{a}) + \nabla f\cdot\vec{a}$

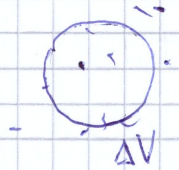
$$\Rightarrow \nabla f\cdot\vec{a} = -f(\nabla\cdot\vec{a}) + \nabla(f\cdot\vec{a})$$

$$\varphi(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\cdot\left[\int\limits_{V'} \nabla'\left(\frac{1}{|\vec{r}-\vec{r}'|}\cdot\vec{P}(\vec{r})\right) dV - \int\limits_{V'} \frac{1}{|\vec{r}-\vec{r}'|}\nabla\cdot\vec{P}(\vec{r})\, dV'\right]$$

protože Gaussova věta

$$\varphi(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\cdot\left[\underbrace{\int\limits_{V'} \nabla\left(\frac{\vec{P}(\vec{r})}{|\vec{r}-\vec{r}'|}\right) dV}_{\text{objemový } Q} - \underbrace{\oint\limits_{S'(V')} \frac{\vec{P}(\vec{r})\cdot\vec{n}}{|\vec{r}-\vec{r}'|}\, dS}_{\text{plošný } Q}\right]$$

MIKRO x MAKRO



$f(x)$ 1D: $\bar{f} = \dfrac{\int f(x)\,dx}{\Delta x}$

3D: $\bar{f} = \dfrac{\int f\, dV}{\Delta V}$ — správně limita, ale fyzikální, tj. nesmí se mi rozbít fyz. vlast.

makro

mikro

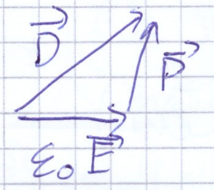
derivace je vůči tomuto „lineární", tj. $\frac{\partial}{\partial x}\overline{f} = \overline{\frac{\partial}{\partial x} f}$

správně by se všude v makros. měly psát čáry, uvažová polarizace

$$\nabla\times\vec{E}=\vec{0} \longrightarrow \nabla\times\overline{\vec{E}}=\vec{0}$$

$$\nabla\cdot\vec{E}=\frac{\rho}{\varepsilon_0} \longrightarrow \nabla\cdot\vec{E}=\frac{\rho+\rho_P}{\varepsilon_0}$$

$$\varepsilon_0\nabla\cdot\vec{E}+\nabla\cdot\vec{P}=\rho$$ ← volného náboje



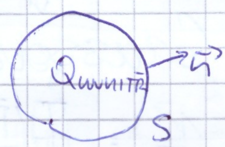
$$\boxed{\nabla\cdot(\underbrace{\varepsilon_0\vec{E}+\vec{P}}_{\vec{D}})=\rho}$$

$\boxed{\vec{D}}$ – vektor el. indukce
(též vektor el. posunutí)

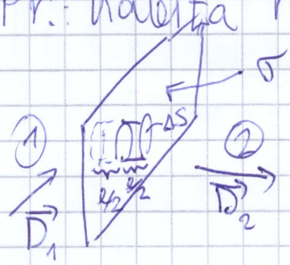
VAKUUM A DIEL.

Gaussův z. z Coulombova z.: $\oint_{S(V)}\vec{E}\cdot\vec{n}\,dS=\frac{Q_{\text{celý uvnitř}}}{\varepsilon_0}\xrightarrow{\dots}$

$\xrightarrow{\dots}\nabla\cdot\vec{E}=\frac{\rho}{\varepsilon_0}$, dá se jít i zpětně $\Rightarrow \oint_{S(V)}\vec{D}\cdot\vec{n}\,dS=Q_{\text{uvnitř}}$



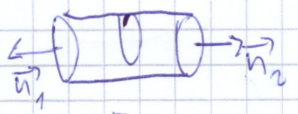
Př.: nabitá rovinná plocha – rozhraní mezi dielektriky



tok $\vec{D}$: $\vec{D}_1\cdot\vec{n}_1\Delta S+\vec{D}_2\cdot\vec{n}_2\Delta S+\text{plášť}=\sigma\Delta S$

$\xrightarrow[\text{plášť}\to 0]{l\to 0}\boxed{(\vec{D}_2-\vec{D}_1)\cdot\vec{n}_2=\sigma}$ – obdobně jak u elstat a $\vec{E}$

$$\boxed{D_{2n}-D_{1n}=\sigma}$$

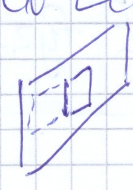


$-\vec{n}_1=\vec{n}_2$

$(\vec{E}_2-\vec{E}_1)\cdot\vec{t}$

$E_{2t}-E_{1t}=0$

a co tečné složky?



cirkulace $\vec{E}$: $\oint\vec{E}\cdot d\vec{l}=\vec{E}_1\cdot\vec{t}_1+\vec{E}_2\cdot\vec{t}_2=\sigma\Delta t$

a co cirk. $\vec{D}$? $\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$

$$\vec{E} = \frac{\vec{D}}{\varepsilon_0} - \frac{\vec{P}}{\varepsilon_0}$$

, když $\oint_l \vec{E} \cdot d\vec{l} = 0$: $E_{t1} = E_{t2}$

$$\frac{D_{t1}}{\varepsilon_0} - \frac{P_{t1}}{\varepsilon_0} = \frac{D_{t2}}{\varepsilon_0} - \frac{P_{t2}}{\varepsilon_0}$$

$$\boxed{D_{t2} - D_{t1} = P_{t2} - P_{t1}}$$

cirkulace $\vec{D}$ obecně po uzav. kř. není nulová, toto nám rozbíjí nenulové $\vec{P}$ ($\Rightarrow$ z $\vec{D}$ neuděláme potenciál)

VLASTNOSTI DIEL.

1. $\vec{P} = \vec{P}(\vec{E})$
- tvrdá diel.: $\vec{P}(\vec{E}) = P_0$
- měkká diel.: $\vec{P}(\vec{E}) = \vec{P}(\vec{E})$
  - pružná (lineární) diel.: $\vec{P}(\vec{E}) \propto \vec{E}$ – model $\vec{p} \uparrow$ $\oplus$ $\ominus$ $E \neq 0$

pružinka

$F_E = F_{PRUŽ.}$

$qE = \varkappa d$

$$P = \underbrace{N\frac{q^2}{\varkappa}}_{\varepsilon_0 \chi \text{ – susceptibilita}} E \Leftarrow p = \underbrace{\frac{q^2}{\varkappa}}_{\beta \text{ – polarizovatelnost}} E \Leftarrow d = \frac{q}{\varkappa} E$$

platí pro $\vec{E} \parallel \vec{P}$:

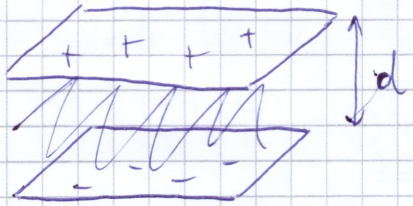
vektorově: $\vec{P} = \varepsilon_0 \chi \vec{E}$

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P} = \varepsilon_0 \underbrace{(1+\chi)}_{\varepsilon_r \text{ – relat. perm.}} \vec{E} =$$

$$= \underbrace{\varepsilon_0 \varepsilon_r}_{\varepsilon \text{ – permitivita dielektrika}} \vec{E}$$

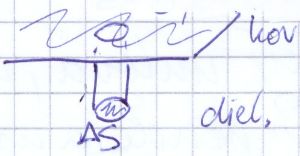
pokud $\vec{E} \nparallel \vec{P}$: $\vec{D} = \overleftrightarrow{\varepsilon} \vec{E}$ – $\overleftrightarrow{\varepsilon}_{ij}$ tenzor ($\overleftrightarrow{\beta}$ tenzor, $\overleftrightarrow{\chi}$ tenzor)

KONDENZ. A DIEL.



vložím mezi desky dielektrikum – – použijeme Gaussův z. s $\vec{D}$

$U = dE$, $\sigma = \frac{Q}{S}$



$$\oint_{S(V_{PLECHOVKY})} \vec{D}\cdot\vec{n}\, dS = Q$$

$$\Rightarrow \underbrace{D_{KOV}}_{\varepsilon E = 0}\cdot\Delta S + D_{DIEL}\,\Delta S = \sigma\Delta S$$

$$\Rightarrow D_{DIEL} = \sigma$$

$$E = \frac{D}{\varepsilon} = \frac{\sigma}{\varepsilon} \Rightarrow U = dE = d\frac{\sigma}{\varepsilon} = \underbrace{\frac{1}{\varepsilon}\cdot\frac{d}{S}}_{1/C}\, Q$$

$$\Rightarrow \boxed{C = \varepsilon\frac{S}{d}} > \varepsilon_0\frac{S}{d}$$

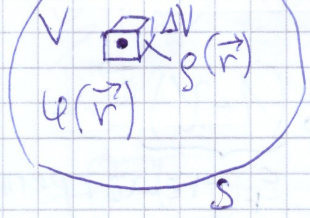
to ale dává smysl: $W = \frac{1}{2}CU^2$, $C_{DIEL} > C_{VAKUUM}$

$\Rightarrow W_{DIEL} > W_{VAKUUM}$ (stejné U)

– dáváme energii „do pružinek" dielektrika

HUSTOTA EL. POLE V DIEL.

pro vakuový kondenz.: $w_e = \frac{1}{2}\varepsilon_0 E^2$



$Q_{\Delta V} = \rho\,\Delta V$

když měním ρ: $\delta W_{\Delta V} = \delta\rho\,\varphi\,\Delta V$

$$\xrightarrow{\Delta V \to 0} \delta W = \int_V \delta\rho\,\varphi\, dV$$

když $\nabla\cdot\vec{D} = \rho$ (G. z.), $\delta\rho = \nabla\cdot\delta\vec{D}$ a $\nabla\cdot(\vec{a}f) = (\nabla\cdot\vec{a})f + \vec{a}\cdot\nabla f$

$$\delta W = \underbrace{\int_V \nabla\cdot(\delta\vec{D}\,\varphi)\,dV}_{G.V.} - \int_V \delta\vec{D}\cdot\underbrace{\nabla\varphi}_{-\vec{E}}\, dV$$

$$= \underbrace{\oint_{S(V)} \underbrace{\delta\vec{D}}_{\sim 1/r^2}\,\underbrace{\varphi}_{\sim 1/r}\cdot\underbrace{\vec{n}\, dS}_{\sim r^2}}_{\sim \frac{1}{r}\to 0} + \int_V \delta\vec{D}\cdot\vec{E}\, dV \xrightarrow{V\to\infty} \delta W = \int_V \delta\vec{D}\cdot\vec{E}\, dV$$

(v diel. očekávám: $w_{e_{DIEL}} = \varepsilon_R w_{e_{vakuum}} = \frac{1}{2}\varepsilon_0 \varepsilon_R E^2 = \frac{1}{2}\vec{E}\cdot\vec{D}$
pro $\vec{E} \parallel \vec{D}$)

$\delta W = \int_V \delta \vec{D} \cdot \vec{E}\, dV = \frac{1}{2}\vec{E}\cdot\vec{D}\cdot V$ — sedí

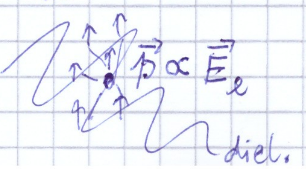
$\varepsilon \int_V \delta \vec{E} \cdot \vec{E}\, dV = \varepsilon \int_V \int_0^E \underbrace{\delta \vec{E} \cdot \vec{E}}_{\text{integrand konst}}\, dV = \varepsilon \int_V \frac{1}{2} E^2\, dV$

DIELEKTRICKÉ VLASTNOSTI LÁTEK

- dielektrika = izolátory
- síla diel. = max. pole, které izol. vydrží, než se prorazí
- diel. homogenní x nehomogenní, izotropní x anizotropní
- diel. tvrdá, měkká (pružná)

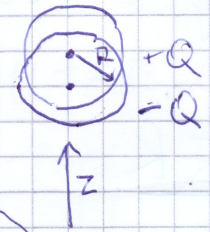
pružné diel.	isot.	anisot.
homog.	χ	$\overleftrightarrow{\chi}$
nehomog.	$\chi(\vec{r})$	$\overleftrightarrow{\chi}(\vec{r})$

- mechanismy polarizace
  - elektronový = vychylují se e^- (p^+)
  - iontový = vychylují se ionty ($Na^+ Cl^-$)
  - (dipólový) orientační = natáčejí se celé molek. (δ^+, δ^-)
  - prostorový náboj na nehomogenitách = skoky na hranicích zrn / domén krystalu
- ε záleží na frekvenci vnějšího pole — při vyšší frekvenci se nestíhají dost rychle přetáčet velké dipóly (prvně zrna, pak molek., ionty a elektrony (protony))
- tzv. lokální pole — dipóly se navzájem ovlivňují (mikroskopicky), musím udělat korekci vnějšího pole



model tzv. Lorentzovy koule – dipólky venku se vyrovnají, uvažuji ty uvnitř

Př.: pole uvnitř diel. koule



$\vec{p} = \begin{pmatrix} 0 \\ 0 \\ p_z \end{pmatrix}$

$$\left.\begin{aligned} Q &= \tfrac{4}{3}\pi R^2 N q \\ \vec{p} &= \vec{d}\, Q \end{aligned}\right\} \Rightarrow \varphi = \frac{1}{4\pi\varepsilon_0}\frac{\vec{d}\cdot\frac{4\pi}{3}R^3 N q\,\vec{r}}{r^3} =$$

$$\varphi = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}\cdot\vec{r}}{r^3}$$

$$= \frac{1}{3\varepsilon_0}\frac{R^3 N q\,\vec{d}\cdot\vec{r}}{r^3} = \frac{1}{3\varepsilon_0}\frac{R^3\,\vec{p}\cdot\vec{r}}{r^3}$$

$$= \frac{1}{3\varepsilon_0}\frac{R^3 p_z \cdot z}{r^3}, \text{ když } R = r$$

$$\varphi = \frac{1}{3\varepsilon_0} p_z \cdot z \quad (\text{splňuje } \Delta\varphi = 0)$$

pole? $\underbrace{\vec{E}_{KOULE}}_{\text{toto pole navíc}} = -\nabla\varphi = \left(0, 0, -\frac{1}{3\varepsilon_0}p\right)$ – homogenní

$$\Rightarrow \vec{E}_{LOKÁLNÍ} = \vec{E}_{VNĚJŠÍ} + \underbrace{\frac{\vec{P}}{3\varepsilon_0}}_{\vec{E}_{KOULE}} + \underbrace{\vec{E}_{UVNITŘ\ KOULE}}_{\text{závisí na látce, pro amorfní nebo kubický / radiálně sym. můžeme } = 0}$$

$$\vec{P} = N\beta\,\vec{E}_{LOKÁLNÍ}, \quad \vec{P} = \varepsilon_0 \overbrace{(\varepsilon_r - 1)}^{\chi}\vec{E}_{VNĚJŠÍ}$$

$$\frac{\vec{P}}{N\beta} = \vec{E}_{VNĚJŠÍ} + \frac{\vec{P}}{3\varepsilon_0}$$ – proč ne $\vec{p}$?

$$\frac{\varepsilon_0(\varepsilon_r - 1)\vec{E}_{VNĚJŠÍ}}{N\beta} = E_{VNĚJŠÍ} + \frac{\varepsilon_0(\varepsilon_r - 1)\vec{E}_{VNĚJŠÍ}}{3\varepsilon_0}$$

$$\frac{\varepsilon_0(\varepsilon_r - 1)}{N\beta} = 1 + \frac{\cancel{\varepsilon_0}(\varepsilon_r - 1)}{3\cancel{\varepsilon_0}}$$

$$\varepsilon_0(\varepsilon_r - 1) = N\beta\left(1 + \frac{\varepsilon_r - 1}{3}\right)$$

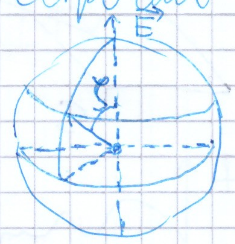
$$\varepsilon_r - 1 = \frac{N\beta}{3\varepsilon_0}(3 + \varepsilon_r - 1)$$

$$\boxed{\frac{N\beta}{3\varepsilon_0} = \frac{\varepsilon_r - 1}{\varepsilon_r + 2}}$$

přibližně pro $\varepsilon_r \approx 1$: $\boxed{\varepsilon_r = 1 + \frac{N\beta}{\varepsilon_0}}$

orientační polarizace: natáčení do směru pole

uvažujeme pole ve směru osy z, energie dipólu v poli závisí na úhlu mezi $\vec{E}$ a $\vec{p}$



počet dipólů stejné energie: (obsazovací číslo)

$$dN_\xi \sim \underbrace{e^{-\frac{W}{kT}}}_{\text{Boltzmann}}, \quad W = -\vec{p}\cdot\vec{E}$$

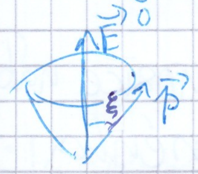
$$\Rightarrow dN_\xi \sim e^{\frac{\vec{p}\cdot\vec{E}}{kT}}, \quad \vec{p}\cdot\vec{E} = p\cdot E\cdot\cos\xi$$

$$\Rightarrow dN_\xi = C' e^{\frac{p\cdot E\cdot\cos\xi}{kT}} \cdot 2\pi R \sin\xi \, R \, d\xi$$

$\Rightarrow$ celkový počet dipólů

$$N = \int_0^\pi dN_\xi = \int_0^\pi C \sin\xi \, e^{\frac{pE\cos\xi}{kT}} d\xi, \quad \text{když } \frac{pE}{kT} = a, \cos\xi = x$$

$$\Rightarrow -\sin\xi \, d\xi = dx$$



potom celkový moment:

$$P = \int_0^\pi p\cos\xi \, C\sin\xi \, e^{a\cos\xi} d\xi = -Cp\int_1^{-1} x e^{ax} dx =$$

$$= -Cp\left[\frac{1}{a} x e^{ax}\Big|_1^{-1} - \int_1^{-1} \frac{1}{a} e^{ax} dx\right] =$$

$$= -Cp\left[\frac{1}{a}(-e^{-a} - e^{a}) - \frac{1}{a^2}(e^{-a} - e^{a})\right]$$

viz výše $C = -Na\frac{1}{e^{-a} - e^{a}}$

$$\Rightarrow P = Na\frac{1}{e^{-a} - e^{a}} p \frac{1}{a}\left[-(e^{a} + e^{-a}) - \frac{1}{a^2}(e^{-a} - e^{a})\right]$$

$$P = Np\left[\frac{e^{a} + e^{-a}}{e^{a} - e^{-a}} - \frac{1}{a}\right] = Np\left[\operatorname{cotanh} a - \frac{1}{a}\right]$$

pro $a \ll 1$: $P \approx$ viz prez.

μ_0 – magnetická permeabilita, $\mu_0 \approx 4\pi \cdot 10^{-7}$ Tm/A

směry jsou dány konvencí – používáme pravotočivé báze

VEKTOROVÝ POTENCIÁL STAC. MAG. POLE

$\vec B = \vec B(\vec r)$

$\vec A = \vec A(\vec r)$ – VEK. POTENCIÁL, $\boxed{\vec B(\vec r) = \nabla\times\vec A(\vec r)}$

protože $\nabla\cdot\vec B = 0$ a $\nabla\times\vec B \neq 0$ lze zavést potenciál takto, není jednoznačné – volí se tzv. kalibrovaný (cejchovaný) vektorový potenciál – pro stac. pole nejčastěji tzv. Coulombova kalibrace: $\boxed{\nabla\cdot\vec A(\vec r) = 0}$, to lze udělat vždy: $\vec B = \nabla\times\vec A = \nabla\times\vec A'$ – 2 vek. pot.

$$\nabla\cdot\vec A(\vec r) = d(\vec r),\quad \nabla\cdot\vec A' = 0$$

$$\nabla\times(\vec A' - \vec A) = 0$$

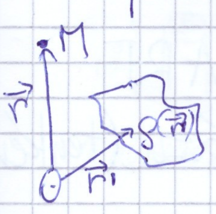
$$\nabla\cdot(\underbrace{\vec A' - \vec A}_{=:\nabla\lambda(\vec r)}) = \nabla\cdot\vec A' - \nabla\cdot\vec A = -d(\vec r)$$

$\Rightarrow \nabla\cdot(\nabla\lambda(\vec r)) = -d(\vec r)$

$\underline{\Delta\lambda(\vec r) + d(\vec r) = 0}$ – když znám $d(\vec r)$ vím jak „posunout" $\vec A$ na $\vec A'$ – tzv. kalibrační transformace

BIOT–SAVARTŮV VZOREC

el. pole:



$$\varphi(\vec r) = \frac{1}{4\pi\varepsilon_0}\int_{V'}\frac{\rho(\vec r')}{|\vec r - \vec r'|}\,dV'$$

$$\Delta\varphi(\vec r) = -\frac{\rho(\vec r)}{\varepsilon_0}$$

mag. pole: $\nabla\times\vec B(\vec r) = \mu_0\vec j(\vec r)$

$$\nabla\times(\nabla\times\vec A) = \mu_0\vec j$$

$$\nabla(\underline{\nabla\cdot\vec A}) - \Delta\vec A = \mu_0\vec j$$

pro kalib. $\vec A$: $\boxed{\Delta\vec A = -\mu_0\vec j}$

po složkách: $\Delta A_x = -\mu_0 j_x$

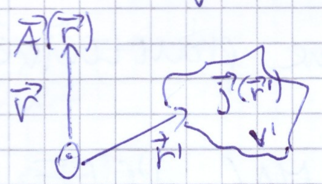
$\Delta A_y = -\mu_0 j_y$

$\Delta A_z = -\mu_0 j_z$

$\frac{1}{\varepsilon_0} \sim \mu_0$

$$\Rightarrow A_x(\vec r) = \frac{\mu_0}{4\pi}\int_{V'} \frac{j_x(\vec r')}{|\vec r - \vec r'|}\, dV' \Rightarrow \boxed{\vec A(\vec r) = \frac{\mu_0}{4\pi}\int_{V'}\frac{\vec j(\vec r')}{|\vec r-\vec r'|}\, dV'}$$

$j_x \sim \varrho$



dV'

$$\vec B(\vec r) = \nabla \times \vec A(\vec r) = \frac{\mu_0}{4\pi} \nabla_{\vec r} \times \int_{V'} \frac{\vec j(\vec r')}{|\vec r - \vec r'|}\, dV' = \frac{\mu_0}{4\pi}\int_{V'} \nabla \times \frac{\vec j(\vec r')}{|\vec r - \vec r'|}$$

$$\left[\nabla \times \frac{\vec j(\vec r')}{|\vec r-\vec r'|}\right]_x = \frac{\partial}{\partial y}\left(j_z(\vec r')\left[(x-x')^2+(y-y')^2+(z-z')^2\right]^{-1/2}\right) -$$

$$- \frac{\partial}{\partial z}\left(j_y(\vec r')\left[(x-x')^2+(y-y')^2+(z-z')^2\right]^{-1/2}\right)$$

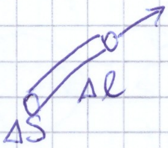
$$= j_z \cdot \left(\frac{-1}{\cancel{2}}\right)\left[(x-x')^2+(y-y')^2+(z-z')^2\right]^{-3/2} \cdot \cancel{2}(y-y')$$

$$- j_y \cdot \left(-\frac{1}{\cancel{2}}\right)\left[(x-x')^2+(y-y')^2+(z-z')^2\right]^{-3/2} \cdot \cancel{2}(z-z')$$

$$= -\underbrace{\left(j_z(y-y') - j_y(z-z')\right)}_{\text{vek. součin}} \cdot \left[(x-x')^2+(y-y')^2+(z-z')^2\right]$$

$$\Rightarrow \nabla \times \frac{j(\vec r')}{|\vec r - \vec r'|} = \frac{\vec j \times (\vec r - \vec r')}{|\vec r - \vec r'|^3}$$

$$\Rightarrow \boxed{\vec B(\vec r) = \frac{\mu_0}{4\pi}\int_{V'} \frac{\vec j(\vec r') \times (\vec r - \vec r')}{|\vec r - \vec r'|^3}\, dV'}$$ BIOT-SAVART v objemu



$I = j\Delta S$, $\vec j \parallel \Delta \vec l$, $\Delta \vec l \perp \Delta S$

$\Delta V = \Delta S \Delta l$ — dosadím

$$\Rightarrow \vec B(\vec r) = \frac{\mu_0}{4\pi}\int \frac{j \Delta S \Delta \vec l \times (\vec r - \vec r')}{|\vec r - \vec r'|}$$

$$\Rightarrow \boxed{\vec B(\vec r) = \frac{\mu_0}{4\pi} I \int_{l'} \frac{d\vec l' \times (\vec r - \vec r')}{|\vec r - \vec r'|^3}}$$ BIOT-SAVART v drátu (tenkém

$d\vec l' \nearrow\!\!\!/\!\!/ \xrightarrow{(\vec r - \vec r')} \cdot M \quad \otimes \vec B(\vec r)$ — sedí s pravou ruko

Př.: $\vec{B} = (0, 0, B_0)^T$

?: $\vec{A}(\vec{r})$

$$\nabla \times \vec{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}, \frac{\partial A_x}{\partial z} - \frac{A_z}{\partial x}, \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

např.: $A_y = B_0 x,\ A_x = A_z = 0 \longrightarrow \vec{A} = (0, B_0 x, 0)$

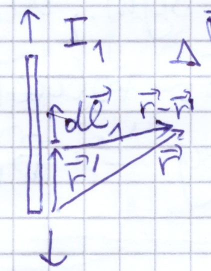
nebo: $A_x = -B_0 y,\ A_y = A_z = 0 \longrightarrow \vec{A} = (-B_0 y, 0, 0)$

nebo: $A_y = \frac{1}{2} B_0 x,\ A_x = -\frac{1}{2} B_0 y,\ A_z = 0 \rightarrow \vec{A} = \frac{1}{2}(-B_0 y, B_0 x, 0)$

?: $\nabla \cdot \vec{A} = 0$

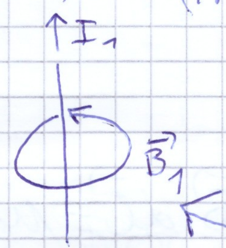
$\nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$ — splňují všechny 3

Př.:

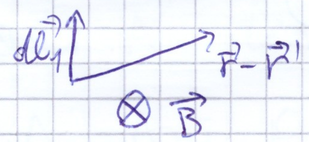


$$\Delta \vec{B}_1 = \left(\frac{\mu_0}{4\pi} I_1\right) \frac{d\vec{l}_1 \times (\vec{r} - \vec{r}')}{(|\vec{r} - \vec{r}'|)}$$

$\Longrightarrow$



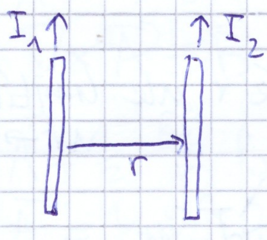
— symetrické



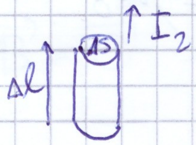
dle Ampera: $\oint_c \vec{B} \cdot d\vec{l} = \mu_0 I$

$$2\pi r B = \mu_0 I_1$$

$$B = \frac{\mu_0}{2\pi} I_1 \cdot \frac{1}{r}$$



$$B_{12} = \frac{\mu_0}{2\pi} I_1 \frac{1}{r}, \quad \vec{f} = \vec{j} \times \vec{B}$$



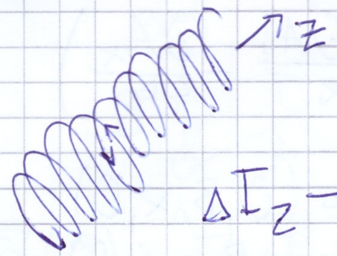
1 působí na 2

$$\vec{F}_{\Delta V} = \Delta V \vec{f} = \Delta l \Delta S \vec{j} \times \vec{B}$$

$$F_{\Delta V} = \Delta l \underbrace{\Delta S j}_{I_2} B \cdot \left(\overset{\alpha = 90°}{\sin \alpha}\right) = I_2 B \Delta l$$

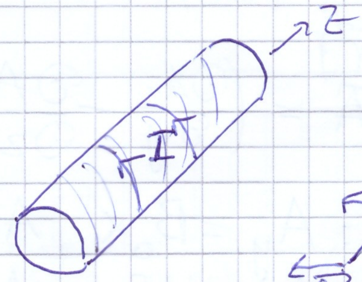
také někdy Ampereův zák. $\boxed{F_{\Delta V} = \frac{\mu_0}{2\pi} I_1 I_2 \frac{1}{r} \Delta l}$ — směr dle viz výš

SOLENOID

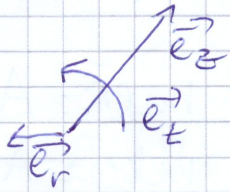


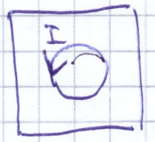
CÍVKA

$\Delta I_z \to 0$



IDEALIZACE





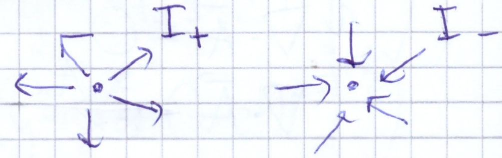
ŘEZ SOLENOIDEM

tečné:

$$\oint_l \vec{B}\cdot d\vec{l} = \mu_0 I = 0$$

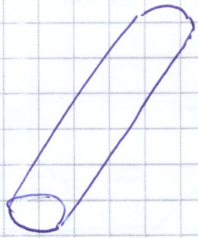
$$B_t \, dl \parallel \Rightarrow B_t = 0$$

radiální:



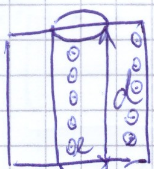
rotač. sym., ale $I_+ \to I_-$ rotací podle jiné osy $\Rightarrow B_R = 0$

SMYČKA

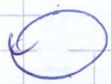


$$\oint_{\text{SMYČKA}} \vec{B}\cdot d\vec{l} = \mu_0 I = 0$$

$$\Rightarrow B_{\text{VENKU}} = 0$$



PODÉL. ŘEZ



SMYČKA

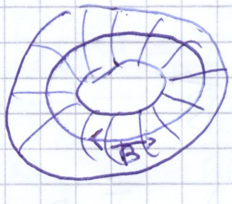
počet závitů /1 cm

$$I_{\text{CELK}} = n d I$$

$$\Rightarrow \overset{\neq 0}{B_z}\, d = \mu_0 I_{\text{CELK}} = \mu_0 n d I$$

$$\Rightarrow \boxed{B_z = \mu_0 \underbrace{nI}_{=:i}}$$

toroid:

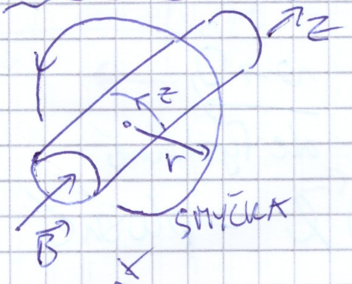


jiné r, ale limitíme $\Delta r \to 0$

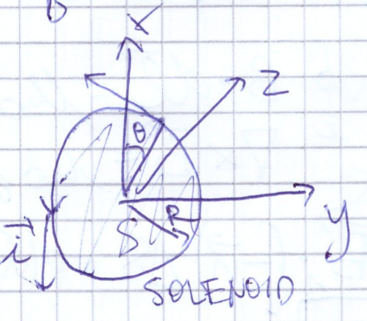
$$2\pi r B = I_{\text{CELK}} = n 2\pi r I \mu_0$$

$$B = nI\mu_0$$

2. $\vec{A}$ v SOLENOIDU



$$\Phi = \int_S \vec{B}\cdot\vec{n}\,dS = \int_S (\nabla\times\vec{A})\cdot\vec{n}\,dS \xrightarrow[\text{analýza}]{\text{vek.}}$$
$$= \oint_{\ell(S)} \vec{A}\cdot d\vec{\ell} \quad - \text{zase rot. sym.}$$



$$\oint_{\ell(S)} \vec{A}\cdot d\vec{\ell} = 2\pi r A = \Phi = \pi R^2 B$$

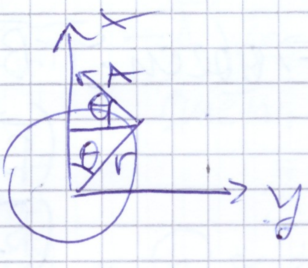
$$A(r) = \frac{1}{2}\frac{R^2}{r} B \,,\ r > R$$

vně nulové pole, ale potenciál nenulový

, ale $\underline{0 < r \leq R}$

$$2\pi r A = \pi r^2 B$$
$$A = \frac{1}{2} r B$$

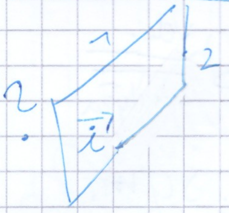
směr $\vec{A}$ stejný jako směr $\vec{i}$



$$A_x = A\sin\theta = A\frac{y}{r} \qquad A_z = 0$$
$$A_y = -A\cos\theta = -A\frac{x}{r}$$
$$\vec{A} = \frac{1}{2} r B\left(\frac{y}{r}, \frac{-x}{z}, 0\right) \quad - \text{pro } 0 < r \leq R$$
$$\vec{A} = \frac{1}{2} B\left(y, -x, 0\right)$$
$$\nabla\times\vec{A} = \vec{B} \quad - \text{sedí}$$

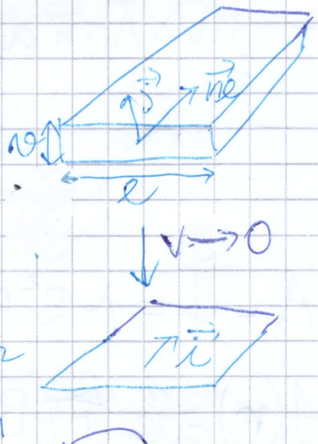
$$\vec{A} = \frac{1}{2}\frac{R^2}{r} B\left(\frac{y}{r}, \frac{-x}{r}, 0\right)$$
$$\vec{A} = \frac{1}{2} R^2 B\left(\frac{y}{r^2}, \frac{-x}{r^2}, 0\right)$$
$$\vec{B} = \vec{0} \quad - \text{sedí}$$

ROZHRANÍ



$$I = (\vec{j}\cdot\vec{n}_e)\, v\, l \xrightarrow{v\to 0}$$ ale fyzikálně

$$\longrightarrow I = (\vec{i}\cdot\vec{n}_e)\, l \quad - \text{plošné}$$



$$\oint_S \vec{B}\cdot d\vec{S} = 0 \qquad \oint_l B\cdot d\vec{l} = \mu_0 I$$

$$\nabla\cdot\vec{B} = 0 \qquad \nabla\times\vec{B} = \mu_0\vec{j}$$

G. plechovka

$$\vec{B}_2\cdot\vec{n}\, dS + \vec{B}_1\cdot(-\vec{n})\,\Delta S + \text{plášť} =$$

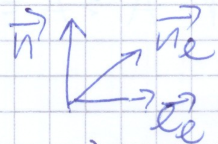
$$\Rightarrow \boxed{B_{2N} = B_{1N}}$$

G. rámeček

$$\vec{B}_2\cdot\vec{e}_e\, l + \vec{B}_1\cdot(-\vec{e}_e)\, l = \mu_0 l\vec{i}$$

$$\Rightarrow B_{2T} - B_{1T} = \mu_0\vec{i} \quad - \vec{i} \text{ kolmo}$$

$$\Rightarrow B_{2T} - B_{1T} = 0 \quad - \vec{i} \text{ rovnob.}$$



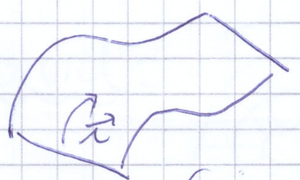
$$\Rightarrow \text{obecně} \quad \vec{B}_2\cdot\vec{e}_e\, l + \vec{B}_1\cdot(-\vec{e}_e)\, l = \mu_0\vec{i}\cdot\vec{n}_e$$

$$(\vec{B}_2 - \vec{B}_1)\cdot\vec{e}_e = \mu_0\,\vec{i}\cdot\vec{n}_e$$

$$(\vec{B}_2 - \vec{B}_1)\cdot\vec{e}_e = \underbrace{\vec{i}\cdot(\vec{n}\times\vec{e}_e)}_{(\vec{i}\times\vec{n})\cdot\vec{e}_e}\,\mu_0$$

$$\Rightarrow \boxed{\vec{B}_2 - \vec{B}_1 = \mu_0\,\vec{i}\times\vec{n}}$$

SPOJITOST VEK. POT. $\vec{A}$



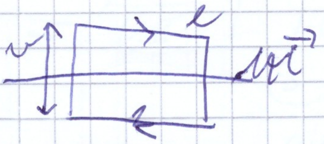
$$\vec{B} = \nabla\times\vec{A}, \quad \text{kal. podm. } \nabla\cdot\vec{A} = 0$$

$$\oint A\cdot\vec{n}\, dS = 0$$

$$\left(\text{analog. } \nabla\cdot\vec{B} = 0 \Rightarrow B_{1N} = B_{2N}\right) \Rightarrow \boxed{A_{1N} = A_{2N}}$$

$$\Phi = \int_S \vec{B}\cdot\vec{n}\, dS$$

$$= \int_S (\nabla\times\vec{A})\cdot\vec{n}\, dS = \oint_{l(S)} \vec{A}\cdot d\vec{l}$$



$$\Phi = \int_S \vec{B}\cdot\vec{n}\, dS \xrightarrow{v\to 0} \underbrace{\vec{B}\cdot\vec{n}\,\underbrace{\Delta S}_{\to 0}}_{\to 0}$$

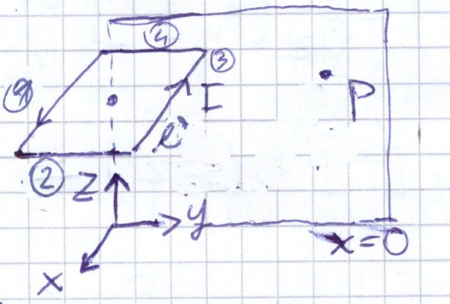
$$\Rightarrow \oint_{l(S)} \vec{A}\cdot d\vec{l} \to 0$$

$$\Rightarrow A_{1t}\cdot l - A_{2t}\cdot l = 0$$

$$\Rightarrow \boxed{A_{1t} = A_{2t}}$$ – potenciál $\vec{A}$ je spoj.

MAGNETICKÉ POLE PROUDOVÉ SMYČKY

1. nepřesné – rovinná pravoúhelníková

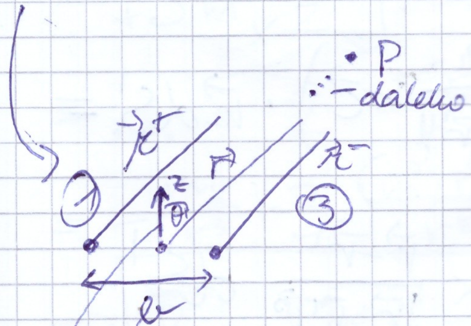


$? \vec{A}(\vec{r})$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} I \oint_{l'} \frac{d\vec{l}}{|\vec{r}-\vec{r}'|}$$ – $\vec{A}$ ve směru $d\vec{l}$

$A_z = 0$ – neteče I

$A_y = 0$ – větve ② a ④ se zruší



$r^+ = r + \frac{b}{2}\sin\theta$

$r^- = r - \frac{b}{2}\sin\theta$

$$\Rightarrow A_x \propto \frac{a}{r^+} - \frac{a}{r^-} = a\left(\frac{1}{r+\frac{b}{2}\sin\theta} - \frac{1}{r-\frac{b}{2}\sin\theta}\right) =$$

$$= a\left(\frac{r-\frac{b}{2}\sin\theta - r - \frac{b}{2}\sin\theta}{(r+\frac{b}{2}\sin\theta)(r-\frac{b}{2}\sin\theta)}\right) \overset{b \ll r}{\approx} a\,\frac{-b\sin\theta}{r^2}$$

$\vec{n} = \vec{e}_z \Rightarrow |\vec{e}_z \times \frac{\vec{r}}{r}| = \sin\theta$

$$\Rightarrow A_x \propto +ab\,\frac{(\vec{n}\times\frac{\vec{r}}{r})_x}{r^2}$$

$$\Rightarrow A_x = \frac{+\mu_0}{4\pi} I ab \frac{(\vec{n}\times\vec{r})_x}{r^3}$$

$I\cdot S =: |\vec{m}|$ – magnetický moment smyčky

$I\cdot S\cdot\vec{n} =: \vec{m}$

na konec:

$$\Rightarrow \boxed{\vec{A} = \frac{\mu_0}{4\pi}\frac{\vec{m}\times\vec{r}}{r^3}}$$

analogické k dipólům

2. korektně – lib. smyčka

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} I \oint_{l'} \frac{d\vec{l}'}{|\vec{r}-\vec{r}'|}$$

$$f(\vec{r}') = \frac{1}{|\vec{r}-\vec{r}'|}$$

$$\rightarrow \oint_{l'} f(\vec{r}')\, d\vec{l}' \;;\; \vec{c} \text{ – konst.}$$

$$\rightarrow \oint_{l'} \vec{c} f(\vec{r}') \cdot d\vec{l}' = \int_{S'} \mathrm{rot}(\vec{c} f(\vec{r}')) \cdot \vec{n}\, dS'$$

$$\mathrm{rot}(\vec{c} f(\vec{r}')) = \nabla \times \left(\vec{c}\, [\quad]^{-1/2}\right)$$

$$f(\vec{r}') = \Big[\overbrace{(x-x')^2 + (y-y')^2 + (z-z')^2}\Big]^{-1/2}$$

$$\left\{\nabla \times \left(\vec{c}\, [\quad]^{-1/2}\right)\right\}_x = \frac{\partial}{\partial y'}\left(c_z [\quad]^{-1/2}\right) - \frac{\partial}{\partial z'}\left(c_y [\quad]^{-1/2}\right) =$$

$$= c_z [\quad]^{-3/2} \cancel{\left(-\tfrac{1}{2}\right)} 2 (y-y') \cancel{(-1)} - c_y [\quad]^{-3/2} (z-z') =$$

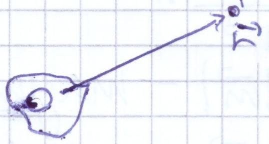
$$= \left\{(\vec{r}-\vec{r}') \times \vec{c}\, \frac{1}{|\vec{r}-\vec{r}'|^3}\right\}_x$$

$$\Rightarrow \int_{S'} \mathrm{rot}(\vec{c} f(\vec{r}')) \cdot \vec{n}\, dS' = \int_{S'} \left(\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|} \times \vec{c}\right) \cdot \vec{n}\, dS' =$$

$$= \int_{S'} \vec{n} \times \frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3} \cdot \vec{c}\, dS' = \vec{c} \cdot \int_{S'} \vec{n} \times \frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3}\, dS'$$

$$\Rightarrow \boxed{\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} I \int_{S'} \vec{n} \times \frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3}\, dS'}$$ vek. pot. smyčky

ELEMENTÁRNÍ SMYČKA



$\vec{r}' \ll \vec{r}$
$\vec{r}' \approx 0$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} I \int_{S'} \vec{n} \times \frac{\vec{r}}{r^3}\, dS' =$$

$$= \frac{\mu_0}{4\pi} I \underbrace{\underbrace{\int_{S'} dS' \vec{n}}_{\text{„plocha" smyčky}} \times \frac{\vec{r}}{r^3}}$$

$=: \vec{m}$ – magnetický moment elementární smyčky

? $\vec{B}(\vec{r})$ — $\vec{B}(\vec{r}) = \nabla \times \vec{A}(\vec{r})$

$$\{\nabla \times \vec{A}\}_x \propto \left\{\nabla \times \left(\vec{m} \times \frac{\vec{r}}{r^3}\right)\right\}_x = \frac{\partial}{\partial y}\left(m_x y [\,]^{-3/2} - m_y x [\,]^{-3/2}\right) - \frac{\partial}{\partial z}\left(m_z x [\,]^{-3/2} - m_x z [\,]^{-3/2}\right), \text{ kde}$$

$$[\,] = (x-x')^2 + (y-y')^2 + (z-z')^2$$

$$= m_x [\,]^{-3/2} + m_x y [\,]^{-5/2}\left(-\tfrac{3}{2}\right) 2y - m_y [\,]^{-5/2}\left(-\tfrac{3}{2}\right) 2yx - m_z [\,]^{-5/2}\left(-\tfrac{3}{2}\right) 2zx + m_x [\,]^{-3/2} + m_x z [\,]^{-5/2}\left(-\tfrac{3}{2}\right) 2z =$$

$$= 2 [\,]^{-3/2} m_x + [\,]^{-5/2}\left[(-3) m_x y^2 + 3 m_y xy + 3 m_z xz + 3 m_x z^2\right]$$

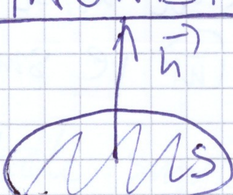
$$= \ldots = 2 \frac{m_x}{r^2} - 3 \frac{m_x r^2}{r^5} + 3 \frac{(\vec{m} \cdot \vec{r}) x}{r^5} =$$

$$= -\frac{m_x}{r^3} + 3 \frac{(\vec{m} \cdot \vec{r}) x}{r^5}$$

$$\Rightarrow \boxed{\vec{B} = \nabla \times \vec{A} = \frac{\mu_0}{4\pi}\left(-\frac{\vec{m}}{r^3} + 3 \frac{(\vec{m} \cdot \vec{r}) \vec{r}}{r^5}\right)}$$

zase analog. k dipólu

MAGNETICKÝ MOMENT

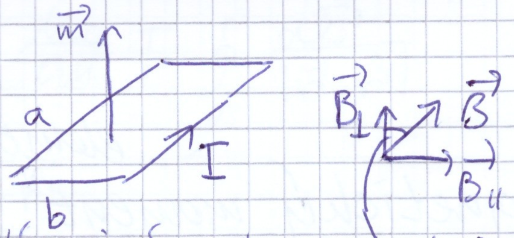


$\vec{m} = I \int_{S'} \vec{n}\, dS$ – mag. moment smyčky (analog. dip. mom.)

pro rovinnou smyčku: $\vec{m} = I S \vec{n}$

PROUDOVÁ SMYČKA VE VNĚJŠÍM MAG. POLI

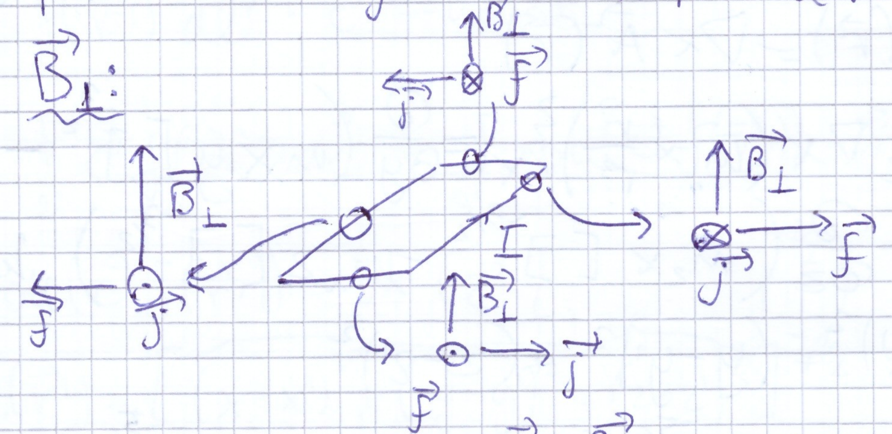
síla:



pravoúhlá rovinná smyčka

α mezi $\vec{m}$ a $\vec{B}$

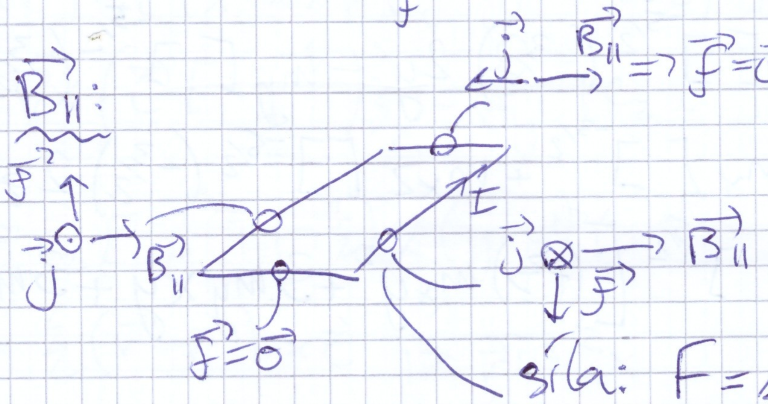
$\vec{B}_\perp$:



$$\vec{f} = \vec{j} \times \vec{B} \quad (Q\vec{v} \times \vec{B})$$

$$\Rightarrow \underline{\sum \vec{F} = \vec{0}}$$

$\vec{B}_\parallel$:



$$(\vec{B} \neq \vec{B}(\vec{r}))$$

síla: $F = \Delta V f = \underbrace{\delta S\, a\, j}_{I} B_\parallel = I a B_\parallel$

moment síly

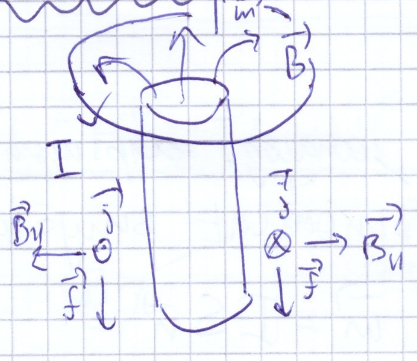
$$M = Fb = \underbrace{abI}_{= SI = m} B_\parallel = m B_\parallel$$

$$\Rightarrow \boxed{\vec{M} = \vec{m} \times \vec{B}}$$

– obecně

analog. $\vec{M} = \vec{p} \times \vec{E}$

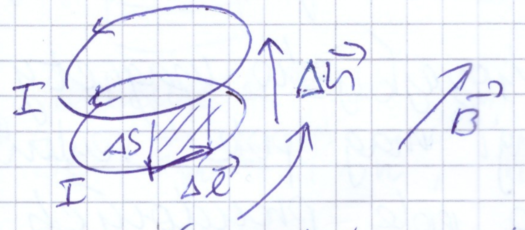
solenoid:



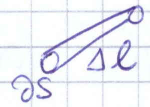
na obou stranách $\vec{f}$ dolů

→ smyčka je vtahována do míst s hustším mag. polem ($\vec{m}$ ve směru pole)

ENERGIE SMYČKY V MAG. POLI



$\vec{B} = \vec{B}(\vec{r})$



$\Delta V_{\Delta l} = \partial S \Delta l$

drát není moc tlustý.

$\vec{j} = j \cdot \frac{\Delta \vec{l}}{\Delta l}$

posunutí o maličko, pomalu

$$\vec{F}_{\Delta l} = \vec{f}\,\Delta V_{\Delta l} = (\vec{j} \times \vec{B})\,\partial S \Delta l = \underbrace{j \partial S}_{=I} \frac{\Delta \vec{l}}{\Delta l} \Delta l \times \vec{B}$$

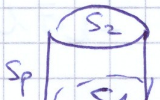
$$\boxed{\vec{F}_{\Delta l} = I \Delta \vec{l} \times \vec{B}}$$

$$\Delta W_{\Delta l} = -\vec{F}_{\Delta l} \cdot \Delta \vec{a} = -I(\Delta \vec{l} \times \vec{B}) \cdot \Delta \vec{a} = -I(\Delta \vec{a} \times \Delta \vec{l}) \cdot \vec{B} = -I \underbrace{\Delta S\, \vec{n}_{\Delta l} \cdot \vec{B}}$$

$= \Phi_{\Delta l}$ – tok $\vec{B}$ plochou ΔS směrem dovnitř

$\longrightarrow$ integrací: tok pláštěm dovnitř,

Víme: $\oint_S \vec{B} \cdot \vec{n}\, dS = 0$, pro válec



$\uparrow \vec{n}$

$$-\int_{S_1} \vec{B} \cdot \vec{n}\, dS + \int_{S_2} \vec{B} \cdot \vec{n}\, dS - \int_{S_{PLÁŠŤ}} \vec{B} \cdot \vec{n}_{\Delta l}\, dS$$

$$\Delta W = -I \int_{S_{PLÁŠŤ}} dS\, \vec{n}_{\Delta l} \cdot \vec{B}$$

$$-\int_{S_1} \vec{B} \cdot \vec{n}\, dS + \int_{S_2} \vec{B} \cdot \vec{n}\, dS = -\frac{\Delta W}{I}$$

$$\Phi_2 - \Phi_1 = -\frac{\Delta W}{I} \longrightarrow \boxed{\begin{aligned} \Delta W &= -I(\Phi_2 - \Phi_1) \\ W &= -I \Phi \end{aligned}}$$

, kde Φ je

tok smyčkou, ve které posouváme, pro malou smyč.

$$\Phi = \vec{B} \cdot \vec{n} \Delta S \Longrightarrow W = -\underbrace{I \Delta S \vec{n}}_{=\vec{m}} \cdot \vec{B} \Longrightarrow \boxed{W = -\vec{m} \cdot \vec{B}}$$

MAG. POLE V LÁTKÁCH (které s mag. p. interagují)

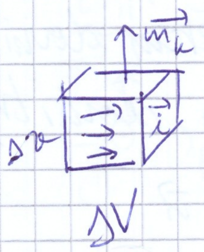
— interagující látky — buď se chovají jako magnety
  nebo ovlivňují mag. pole „venku"

— pole v látkách chápeme jako pole proudových smyček (analog. dipólu v dielek.)

$\vec{M}$ — vektor magnetizace, $\vec{M} = \vec{M}(\vec{r})$ (analog. $\vec{P}$)
— magnetický moment jednotky objemu materiálu

$$\boxed{\begin{aligned}\vec{M}(\vec{r}) &= \lim_{\Delta V \to 0} \frac{\vec{m}_{\Delta V}}{\Delta V} \\ \vec{m}_{\Delta V} &= \int_{\Delta V} \vec{M}(\vec{r})\, dV\end{aligned}}$$

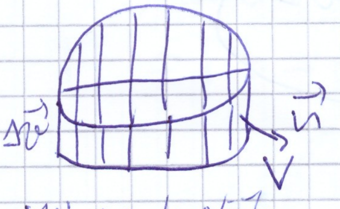
— makroskopická vlastnost látek



kostička látky

$$\vec{m} = I S \vec{n}$$
$$I_k = \vec{i}_k \Delta v$$
$$\vec{m}_k = I_k \Delta S_\Delta \vec{n}_k$$

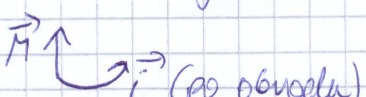
$$\vec{m} = \sum_k \vec{m}_k = I\vec{n} \underbrace{\sum_k \Delta S}_{=S}$$

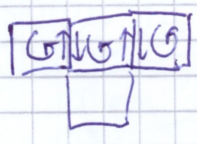


puk na kostičky

$$\vec{M} = \frac{\vec{m}}{V} = \frac{I\vec{n}}{\Delta v}, \quad M = \frac{i \Delta v S}{S \Delta v} = i$$

$$\Rightarrow \boxed{\vec{M} = \vec{i} \times \vec{n}}$$





kostičky

jediné, kde se I nevyruší je na povrchu

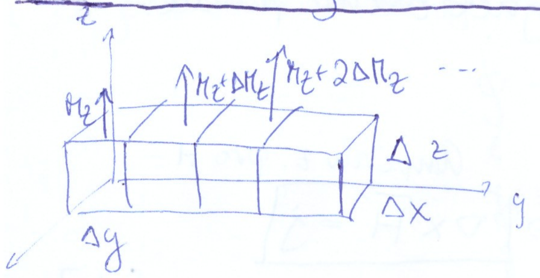
$\vec{i}_{MAG}$ — magnetizační proudová hustota, vázaná na materiál minule: kus látky s magnetizací $\vec{M}$

$\hookrightarrow$ totéž, jako kdyby po tělese tekl proud

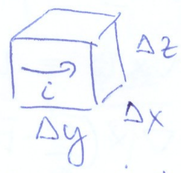
označuje se jako i_m.

Je $\vec{i}_m = \vec{M} \times \vec{n}$, $\vec{n}$ je vnější normála k povrchu tělesa.

Proměnná magnetizace v tělese



jednu kostičku máme jako



$i = M_z$.

v další je $i + \Delta i = M_z + \Delta M_z$

atd.

Celkový proud je

na první kostičce $I = i\,\Delta z$

na druhé kostičce $I + \Delta I = (i + \Delta i)\,\Delta z = (M_z + \Delta M_z)\,\Delta z$

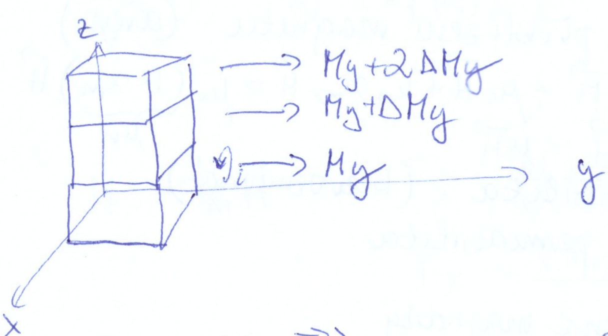
Pohled shora

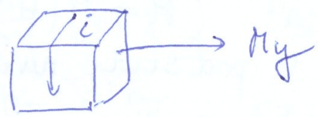
na rozhraní teče proud ΔI.

Je tedy
$$j = \frac{\Delta I}{\Delta z \Delta y}, \quad \Delta M_z = \frac{\partial M_z}{\partial y}\,\Delta y$$

$$\Delta I = \Delta M_z\,\Delta z = \frac{\partial M_z}{\partial y}\,\Delta y\,\Delta z,$$

$$j = \frac{\partial M_z}{\partial y} \quad \text{směru } \vec{e}_x.$$





zopakujeme a dostaneme

$$j = \frac{\partial M_z}{\partial z}, \quad \text{směru } -\vec{e}_x$$

$$j_x = \frac{\partial M_z}{\partial y} - \frac{\partial M_y}{\partial z},$$

$j_x = (\nabla \times \vec{M})_x \longrightarrow$ můžeme zobecnit

$$\boxed{j_m = \nabla \times \vec{M}}$$

(analogie v elstat: $\sigma_p = P_n$ $\quad \rho_p = -\nabla \cdot \vec{P}$)

$\nabla \cdot \vec{B} = 0$ $\quad \oint_S \vec{B} \cdot \vec{n}\, dS = 0$ $\quad \nabla \times \vec{B} = \mu_0 \vec{j}$

$\oint_l \vec{B} \cdot d\vec{l} = \mu_0 I$ — VE VAKUU

musíme vzít navíc v úvahu magnetizační proud.

$$\nabla \times \vec{B} = \mu_0 \vec{j} + \mu_0 \vec{j}_m$$

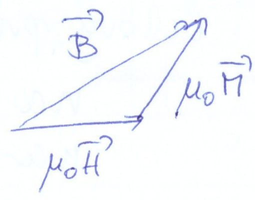
$$\nabla \times \vec{B} = \mu_0 \vec{j} + \mu_0 \nabla \times \vec{M}$$

Ampérův z. pro H

$$\nabla \times \underbrace{\left(\frac{\vec{B}}{\mu_0} - \vec{M}\right)}_{\vec{H}} = \vec{j} \longrightarrow \boxed{\nabla \times \vec{H} = \vec{j}}$$

$\vec{H}$ — vektor magnetické intenzity — $[A/m]$

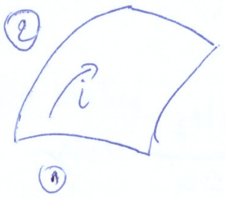
$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$ $\quad \vec{B} = \mu_0 \vec{H} + \mu_0 \vec{M}$



$\nabla \times \vec{H} = j \longrightarrow \boxed{\oint_l \vec{H} \cdot d\vec{l} = I}$

Příklad. Spojitost/nespojitost $\vec{H}, \vec{B}$

nasadíme Gaussovu plechovku $\quad B_{1n} = B_{2n}$



$\otimes i$ $\quad \oint_l \vec{H} \cdot d\vec{l} = H_{1t} \Delta l - H_{2t} \Delta l = i \Delta l$

$H_{1t} - H_{2t} = i$

V obecním směru $\quad H_{1t} - H_{2t} = \vec{n} \times \vec{i}$ - stejné jako $\vec{B}$

pro nulový proud se zachovávají tečné složky, jinak ne

Pozn. víme $\nabla \vec{B} = 0$. Z $\vec{B} = \mu_0 \vec{H} + \mu_0 \vec{M}$ máme $\nabla \cdot \vec{H} + \nabla \cdot \vec{M} = 0$.

Typy magnetik

· slabá $\quad M = M(H) = \chi_m H$ $\quad \chi_m = \text{konst}$ obecně tenzor

v podstatě nereagují na přiblížení magnetu (dřevo)

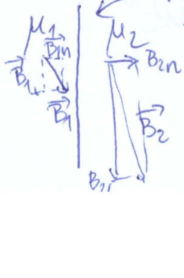
· $\vec{M} = \chi_m \vec{H}$ $\quad \vec{B} = \mu_0 \vec{H} + \mu_0 \vec{M} = \mu_0 \vec{H} + \mu_0 \chi_m \vec{H} = \mu_0 \underbrace{(1 + \chi_m)}_{\mu_r} \vec{H}$

$= \mu_0 \mu_r \vec{H} = \mu \vec{H}$

χ_m – magnetická susceptibilita (bezrozměrná)

$\mu_r = 1 + \chi_m$ – relativní permeabilita

μ – permeabilita



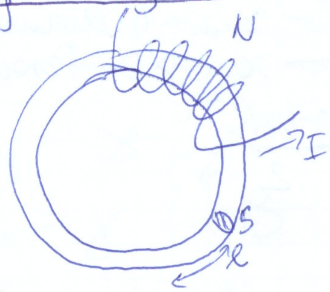
rozhraní mezi dvěma magnetiky

$\vec{B}_{1n} = \vec{B}_{2n}$ $\quad \vec{H}_{1t} = \vec{H}_{2t} \longrightarrow \frac{B_{1t}}{\mu_1} = \frac{B_{2t}}{\mu_2}$

$B_{2t} = B_{1t} \frac{\mu_2}{\mu_1}$ $\quad$ např. $\mu_1 = \mu_0$ $\quad \mu_2 = 10^3$

$\Longrightarrow \vec{B}_2$ skoro rovnoběžné s $\vec{B}_{2t}$

Magnetický obvod



jako prstýnek
navineme N závitů s proudem I

$\oint_l \vec{B}\, d\vec{l}$ zavedeme $\Phi = BS$
mag. tok

$\vec{B}$ jen uvnitř magnetika a $\vec{B} \parallel d\vec{l}$

$$\oint_l \vec{H}\cdot d\vec{l} = NI \quad \rightarrow \text{můžeme psát bez šipek}$$

$$\oint_l H\,dl = \oint_l \frac{B}{\mu}\,dl = \oint_l \frac{\overset{\text{konst.}}{\Phi}\,dl}{S\mu} = \Phi \underbrace{\oint_l \frac{dl}{S\mu}}_{R_n} = \Phi R_n$$

R_n – magnetická reluktance

$NI = \mathcal{E}_m$ – magnetomotorická síla

Dostáváme $\boxed{\mathcal{E}_m = \Phi R_m}$ Hopkinsonův zákon

Dělení látek podle magnetických vlastností

- diamagnetické $\mu_r < 1$ měď, zlato, zinek
- paramagnetické $\mu_r > 1$ hliník, titan, platina
- látky s mag. uspořádáním
  - feromagnetické $\mu_r \gg 1$ železo, kobalt, nikl, neodym
  - anti-feromagnetické
  - ferimagnetické

Síla na magnetikum

$\vec{m} = V\vec{M}$

$W = -(\vec{m}\cdot\vec{B})$

$$F_x = -\frac{\partial W}{\partial x} \qquad F_x = \frac{\partial}{\partial x}(\vec{p}\cdot\vec{E})$$

$$F_x = \frac{\partial}{\partial x}(\vec{m}\cdot\vec{B}) \qquad F_x = (\vec{p}\cdot\nabla)E_x$$

$$F_x = (\vec{m}\cdot\nabla)B_x \qquad \nabla\times\vec{E} = 0$$

$$\nabla\times\vec{B} = 0 \leftarrow \text{chceme}$$

$M = \chi_m H$

$$F_x = m_x\frac{\partial B_x}{\partial x} + m_y\frac{\partial B_y}{\partial y} + m_z\frac{\partial B_z}{\partial z}$$

$$F_x = \frac{V\chi_m}{\mu}\left(B_x\frac{\partial B_x}{x} + B_y\frac{\partial B_y}{\partial x} + B_z\frac{\partial B_z}{\partial x}\right)$$

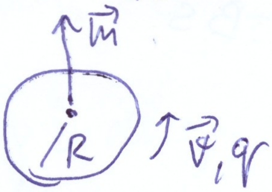
$$= \frac{V\chi_m}{2\mu}\frac{\partial}{\partial x}\underbrace{\left(B_x^2 + B_y^2 + B_z^2\right)}_{B^2} = \frac{V\chi_m}{2\mu}\cdot\frac{\partial}{\partial x}(B^2)$$

$$\vec{F} = \chi_m\frac{V}{2\mu}\nabla B^2$$

MAG. VLASTNOSTI LÁTEK

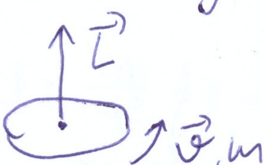
$\vec{M} = \chi_m \vec{H}$ – χ_m vlastnost látky

- $\chi_m > 0$ param.
- $\chi_m < 0$ diam.
- $\chi_m \gg 0$ ferom.



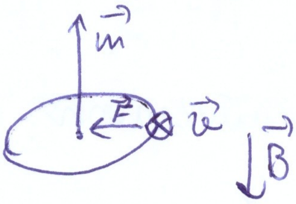
$$m = I \cdot S = q \frac{\omega}{2\pi} \pi R^2 = \frac{q\omega R^2}{2}$$

z mechaniky:



$$L = Rmv = R^2 m\omega$$

$$\frac{m}{L} = \frac{q}{2m} =: \gamma \text{ – GYROMAGNETICKÝ POMĚR}$$



$\vec{F} = q\vec{v} \times \vec{B}$ – radiálně dovnitř

$\vec{B}$ antipar. k $\vec{m}$:

$B = 0$: $F_{ODSTŘ} = F_{COULOMB}$

$$m\omega^2 R = \frac{mv^2}{R}$$

dosaz.

$B \neq 0$: $F_{ODSTŘ} = F_{COUL} + F_{LORENTZ}$

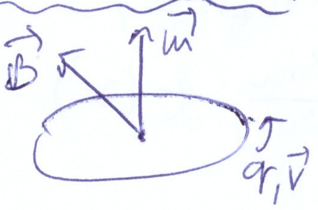
$$m\omega'^2 \cancel{R} = m\omega^2 \cancel{R} + q\omega' \cancel{R} B$$

$$\frac{\omega'^2 - \omega^2}{\omega'} = \frac{qB}{m}$$

$$\Delta\omega := \frac{(\omega' - \omega)\overbrace{(\omega' + \omega)}^{\approx 2\cancel{\omega'}}}{\cancel{\omega'}} = \frac{qB}{m}$$

$\Delta\omega = \frac{q}{2m} B$ – vychází gyrom. poměr

$\vec{B}$ lib. k $\vec{m}$:



$$\vec{M} = \vec{m} \times \vec{B}$$

2. v. imp.: $\frac{d\vec{L}}{dt} = \vec{M}$

$$\frac{d\vec{L}}{dt} = \vec{m} \times \vec{B} = \gamma \vec{L} \times \vec{B}$$, BÚNO $\vec{B} = (0, 0, B)$

$$\left.\begin{aligned} \dot{L}_x &= \gamma L_y B \\ \dot{L}_y &= \gamma L_x B \end{aligned} \quad \dot{L}_z = 0 \right\} \Rightarrow \text{precese kolem } \vec{B}$$

jak rychle?

$$\ddot{L}_x = \gamma^2 L_x B^2$$

$$\ddot{L}_x + \Omega^2 L_x = 0 \longrightarrow \Omega = \frac{qB}{2m}$$ — LARMOROVA FREKVENCE

6. Kvazistacionární mag. pole

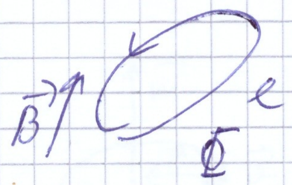
$\vec{E} = \vec{E}(\vec{r}, t)$

$\vec{B} = \vec{B}(\vec{r}, t)$ – pomalé změny v čase

$f \sim 50\,Hz, 100\,Hz$

– platí zákony stac. mag. s malými změnami (jako dobrá aproximace), a to s dodatkem

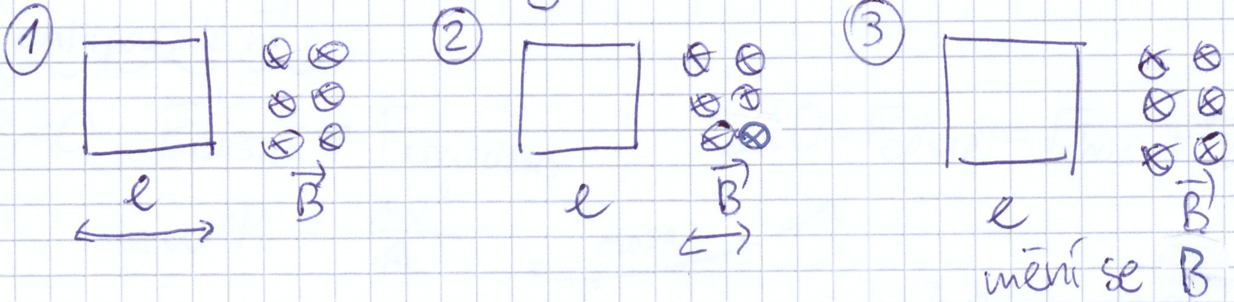
FARADAYŮV ZÁKON ELMAG. INDUKCE



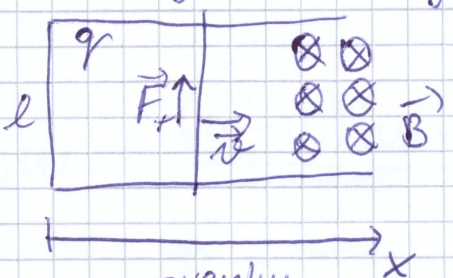
$$\mathcal{E}_F = -\frac{d\Phi}{dt}$$

$\mathcal{E}_F$ – elmag. napětí

tři možnosti změny Φ:



pozorujeme vždy stejně



$\vec{F} = q\vec{v} \times \vec{B}$

$F_{LOR} = qvB$

zvenku „vtištěná"

$$E^* = \frac{F_{LOR}}{q} = vB$$

$$\mathcal{E} = E^* l = lvB$$

sedí

$$\Phi = BS = Blx$$

ELEKTROMAGNETICKÁ INDUKCE

$$\mathcal{E}_F = -\frac{d\Phi}{dt}$$

$$\Phi = \int_S \vec{B}\cdot\vec{n}\,dS \qquad \vec{B} = \vec{B}(\vec{r},t)$$

$$\oint_\ell \vec{E}\cdot d\vec{\ell} = -\frac{d}{dt}\left(\int_S \vec{B}\cdot\vec{n}\,dS\right)$$

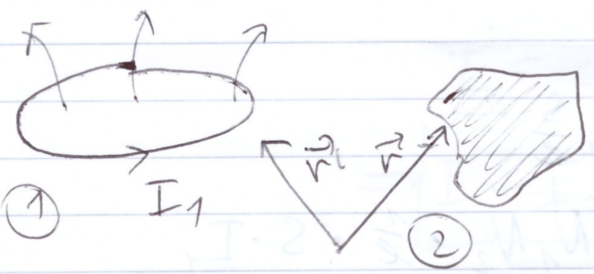
konst. v prostoru
konst. v čase

$$\longrightarrow -\int_S \frac{\partial \vec{B}}{\partial t}\cdot\vec{n}\,dS = \oint_\ell \vec{E}\cdot d\vec{\ell}$$

$$\int (\nabla\times\vec{E})\cdot\vec{n}\,dS = -\int_S \frac{\partial\vec{B}}{\partial t}\,\vec{n}\,dS$$

$$\boxed{\nabla\times\vec{E} + \frac{\partial\vec{B}}{\partial t} = \vec{0}}$$

INDUKČNOST - VZÁJEMNÁ, VLASTNÍ



$$\vec{B}_1(\vec{r}) = \frac{\mu_0}{4\pi} I_1 \oint_{\ell_1} \frac{d\vec{\ell}_1\times(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}$$

$$\Phi_{21} = \int_{S_2} \vec{B}_1\cdot\vec{n}\,dS \propto I_1$$

$\Phi_{21} = L_{21} I_1$, L_{21} - vzájemná indukčnost smyček

$$\left(\Phi = \int_S \vec{B}\cdot\vec{n}\,dS = \int_S (\nabla\times\vec{A})\,\vec{n}\,dS = \oint_{\ell(S)} \vec{A}\cdot d\vec{\ell}\right)$$

$$\Phi_{21} = \oint_{\ell_2} \vec{A}_1\cdot d\vec{\ell}_2, \quad \vec{A}_1 = \frac{\mu_0}{4\pi} I_1 \oint_{\ell_1} \frac{d\vec{\ell}_1}{|\vec{r}-\vec{r}'|^2}$$

$$\Phi_{21} = I_1 \cdot \underbrace{\frac{\mu_0}{4\pi}\oint_{\ell_1}\oint_{\ell_2} \frac{d\vec{\ell}_1\cdot d\vec{\ell}_2}{|\vec{r}-\vec{r}'|^2}}_{L_{21}}$$

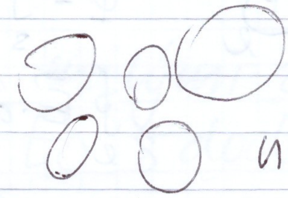
r' běží po ℓ_1
r běží po ℓ_2

vlastní indukčnost

I

$$\Phi = L \cdot I \; (= L_{11} \cdot I)$$

$$\mathcal{E}_F = -\frac{d\Phi}{dt}$$



n smyček

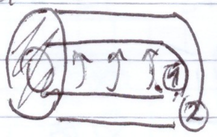
$$\Phi_j = \sum_{k=1}^{n} \Phi_{jk} \qquad \mathcal{E}_{F_j} = -\frac{d\Phi_j}{dt}$$

Př: solenoidy

$S_1 = S_2$

N_1

N_2



$$B = \mu_0 i$$

$$B_1 = \mu_0 I_1 \frac{N_1}{l}$$

$$\Delta\Phi_{21} = S \cdot B_1 = \mu_0 I_1 N_1 \frac{S}{l}$$

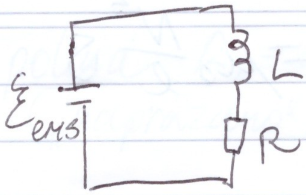
$$\Phi_{21} = N_2 \cdot S \cdot B_1 = \underbrace{\mu_0 N_1 N_2 \cdot \frac{1}{l} \cdot S}_{L_{21}} \cdot I_1$$

OBVODY S INDUKČNOSTÍ

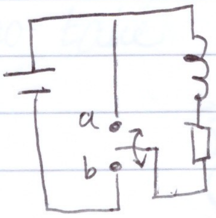
pro $\underbrace{\quad}_{}$ magstat. obvody Kirchhoffova pravidla

① $\sum I_k = 0$ $\qquad\qquad \xrightarrow{\text{doplníme}}$ $\qquad \sum I_k = 0$

② $RI = \mathcal{E}_{ems}$ $\qquad$ pro kvazstac. mag. $\qquad RI = \mathcal{E}_{ems} + \mathcal{E}_F$



$$\mathcal{E}_F = -L\frac{dI}{dt}$$



b); ustálený stav $\frac{dI}{dt} = 0$, $\mathcal{E}_F = 0$

$$RI_0 = \mathcal{E}_{ems}'$$

$$\Rightarrow I_0 = \frac{\mathcal{E}_{ems}}{R}$$

přepojení na a); pak $RI = -L\frac{dI}{dt}$

v čase $t = 0$: $I = I_0$

řešíme dif. rci.: $\frac{dI}{I} = \frac{-R}{L}dt$

$$\ln I = -\frac{R}{L}t + C$$

$$I = e^{-\frac{R}{L}t + C} \quad , e^C = I_0$$

$$\Rightarrow \boxed{I = I_0 e^{-\frac{R}{L}t}} \text{ – tlumení}$$

to sedí protože ohmické ztráty

$$dW = RI^2 dt$$

$$W = \int_0^\infty RI^2 dt = I_0^2 R \int_0^\infty e^{-2\frac{R}{L}t} dt$$

$$W = \left[-I_0^2 R \frac{L}{2R} e^{-\frac{2R}{L}t}\right]_0^\infty$$

$$\boxed{W = \frac{1}{2} L I_0^2} \qquad \boxed{W = \frac{1}{2} \Phi I_0}$$

pro solenoid (viz př. výše): $L=\mu_0 \frac{N^2}{l} S$

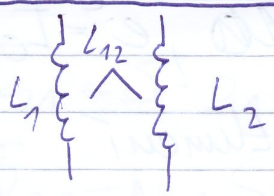
$$W=\frac{1}{2}\mu_0 \frac{N^2}{l} S I_0^2$$

$$W=\frac{1}{2}\underbrace{\mu_0 \frac{N}{l} I_0}_{B}\ \underbrace{\frac{N}{l} I_0}_{H}\ \underbrace{lS}_{V}$$

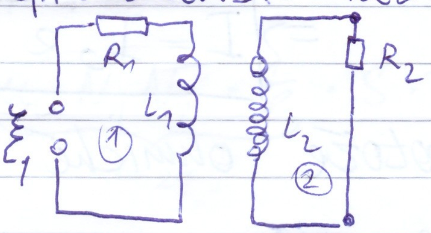
$$\boxed{w_e=\frac{1}{2}BH} \longrightarrow \boxed{w_e=\frac{1}{2}\vec{B}\cdot\vec{H}}$$

$\Rightarrow W=\frac{1}{2}\int A j \, dV$, neboť $\vec{B}=\nabla\times\vec{A}$.

INDUKČNĚ VÁZANÉ OBVODY



– např. transformátor



$$1:\ \mathcal{E}_1+\mathcal{E}_{1F}=R_1 I_1$$
$$2:\ 0+\mathcal{E}_{2F}=R_2 I_2$$

$$\mathcal{E}_{jF}=-L_{j1}\dot{I}_1-L_{j2}\dot{I}_2$$

pokud $R_2=0$: $\mathcal{E}_{2F}=0$
(„nakrátko") $-L_{21}\dot{I}_1-L_{12}\dot{I}_2=0$

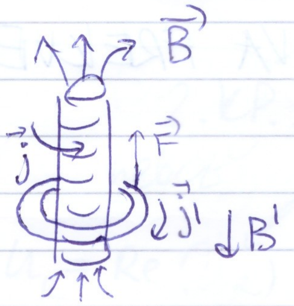
$$\Rightarrow \left|\frac{I_2}{I_1}\right|=\left|\frac{L_{21}}{L_{12}}\right|=\frac{N_1}{N_2}$$

viz dříve

pokud $R_2 = \infty$: $I_2 = 0 \rightarrow |\mathcal{E}_{20}| = |L_{12}\dot{I}_1| = |\mathcal{E}_1 \frac{L_{12}}{L_{11}}|$
(„naprázdno") $R_1 \doteq 0 \rightarrow \mathcal{E}_1 - L_{11}\dot{I}_1 = 0$

$$\left|\frac{\mathcal{E}_{20}}{\mathcal{E}_1}\right| \doteq \left|\frac{L_{12}}{L_{11}}\right| = \frac{N_2}{N_1}$$

nebo také



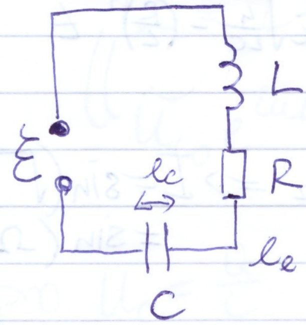
$\uparrow \vec{F}_L$

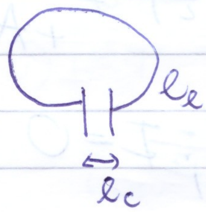
$\odot\, j' \longrightarrow \vec{B}^E$ - pouze kolmé
viz dříve

$\downarrow B'$

na tomto principu generátory a motory - viz prezentace

OBVOD S INDUKČNOSTÍ A KAPACITOU





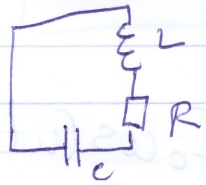
$$\oint \vec{E}\cdot d\vec{l} = \int_{l_e} \vec{E}\cdot d\vec{l} + \underbrace{\int_{l_c} \vec{E}\cdot d\vec{l}}_{= +U_c} = 0$$

$$R\cdot I = \mathcal{E} + \mathcal{E}_F + \underbrace{\int_{l_e} \vec{E}\cdot d\vec{l}}_{= -U_c}$$

$$R\cdot I + U_c = \mathcal{E} + \mathcal{E}_F \leftarrow = \underbrace{-L\frac{dI}{dt}}_{=: U_L}$$ - napětí cívky

$$\boxed{R\cdot I + U_c + U_L = \mathcal{E}}$$

Př.: oscilační obvod



$$RI + \frac{Q}{C} + L\frac{dI}{dt} = 0 \qquad / \frac{d}{dt}$$
$$R\dot{I} + \frac{1}{C}I + L\cdot\ddot{I} = 0$$

$\ddot{I} + \frac{R}{L}\dot{I} + \frac{1}{LC} I = 0$ – tlumené kmity (viz mech.

$(\ddot{x} + \gamma \dot{x} + \Omega_0^2 x = 0)$

tření, vlastní frekvence

$\Omega_0^2 = \frac{1}{LC}$ – THOMPSONOVA FREKVENCE

$\Omega_0 = \frac{1}{\sqrt{LC}}$

$I \propto e^{\alpha t}$: $\alpha^2 + \frac{R}{L}\alpha + \frac{1}{LC} = 0$

$D < 0$: $\alpha_{1,2} = -\frac{R}{2L} \pm \frac{i}{2}\sqrt{\frac{4}{LC} - \left(\frac{R}{L}\right)^2}$

($D \geq 0$: „nadkritický" tlumené – neosciluje)

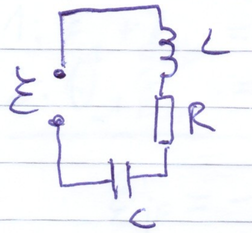
$I = A_1 e^{-\frac{R}{2L}t} \cdot e^{i\sqrt{\frac{4}{LC}-\left(\frac{R}{L}\right)^2}\, t} + A_2 e^{-\frac{R}{2L}t} e^{-i\sqrt{\frac{4}{LC}-\left(\frac{R}{L}\right)^2}\, t}$

pro poč. podm. $t = 0$, $I = 0$: $A_1 = -A_2 \Rightarrow I = \sin(\sqrt{\ }$
$= \sin(\Omega t$

pro $R \ll L$: $\Omega = \Omega_0$ – není tlumení

RLC OBVODY SE STŘÍDAVÝM PROUDEM

$\mathcal{E} = \mathcal{E}_0 \cos(\omega t + \varphi_0)$



$I = I_0 \cos(\omega t + \varphi_0) =$
$= I_0 \frac{1}{2}\left(e^{i(\omega t+\varphi_0)} + e^{-i(\omega t+\varphi_0)}\right) =$
$= \frac{1}{2} I_0 \cdot e^{i\varphi_0} \cdot e^{i\omega t} + \frac{1}{2} I_0 e^{-i\varphi_0} \cdot e^{-i\omega t}$

zavedeme komplexní amplitudu: $\tilde{I}_0 = I_0 e^{i\varphi_0}$, $\tilde{I}_0^{*} = I_0 e^{-i\varphi_0}$ komp. sdruž.

(též $\hat{I}_0, \hat{I}$)

$\tilde{I} = \tilde{I}_0 e^{i\omega t}$

$I = Re(\tilde{I}) = I_0 \cos(\omega t + \varphi_0)$

užitečné, neboť 1. KP.: $\sum_j I_j = 0 \xrightarrow{\text{komp.}} \boxed{\sum_j \tilde{I}_{0j} = 0}$,

neboť $\sum_j I_j = \sum_j Re(\hat{I}_j) = Re(\sum_j \tilde{I}_{0j} e^{i\omega t}) =$

$= Re(e^{i\omega t} \sum_j \tilde{I}_{0j}) = 0$

2. KP.: $U_R = R \cdot I \xrightarrow{\text{komp.}} \boxed{\tilde{U}_{R_0} = R \cdot \tilde{I}_0}$

neboť $\tilde{U}_R = \tilde{U}_{R_0} e^{i\omega t} = R \cdot \tilde{I}_0 e^{i\omega t}$

$\tilde{U}_{R_0} = R \cdot \tilde{I}_0$

$U_R = Re(\tilde{U}_R) = Re(R\tilde{I}_0) = RI$

$\Rightarrow \boxed{Z_R := R}$ – resistivní impedance, (resistance)

pro $U_L = L \frac{dI}{dt}$

$$\left.\begin{aligned} \tilde{U}_L &= L\frac{d\tilde{I}}{dt} = i\omega L \tilde{I}_0 e^{i\omega t} \\ &\ \| \\ &\tilde{U}_{L_0} e^{i\omega t} \end{aligned}\right\} \Rightarrow \tilde{U}_L = \boxed{\underbrace{i\omega L}_{=: Z_L}} \tilde{I}_0$$

– induktivní impedance (induktance)

pro $U_C = \frac{Q}{C}$

$\dot{U}_C = \frac{I}{C}$

$\frac{d}{dt}(\tilde{U}_0 e^{i\omega t}) = \frac{\tilde{I}_0 e^{i\omega t}}{C}$

$i\omega \tilde{U}_0 e^{i\omega t} = \frac{1}{C} \tilde{I}_0 e^{i\omega t}$

$\tilde{U}_0 = \boxed{\underbrace{\frac{1}{i\omega C}}_{=: Z_C}} \tilde{I}_0$ – kapacitní impedance (kapacitance)

$\xRightarrow{\text{CELKEM}} \boxed{\tilde{\mathcal{E}}_0 = Z_L \tilde{I}_0 + Z_R \tilde{I}_0 + Z_C \tilde{I}_0}$

Př.: výkon AC

$$P = U \cdot I = \tfrac{1}{2} U_0 \left[e^{i(\omega t + \varphi_U)} + e^{-i(\omega t + \varphi_U)} \right] \cdot \tfrac{1}{2} I_0 \left[e^{i(\omega t + \varphi_I)} + e^{-i(\omega t + \varphi_I)} \right]$$

$$= \tfrac{1}{4} U_0 I_0 \left[\underbrace{e^{i(2\omega t + \varphi_U + \varphi_I)} + e^{-i(2\omega t + \varphi_U + \varphi_I)}}_{\approx \cos(2\omega t)} + \underbrace{e^{i(\varphi_U - \varphi_I)} + e^{-i(\varphi_U - \varphi_I)}}_{= 2\cos(\varphi_U - \varphi_I)} \right]$$

$$\langle P \rangle_t = \frac{1}{\underset{\text{perioda}}{T}} \int_t^{t+T} P(t)\, dt = \frac{U_0 I_0}{2} \cdot \underbrace{\cos(\varphi_U - \varphi_I)}_{\text{účinník}}$$

$$I_{EF} = \frac{I_0}{\sqrt{2}}, \quad U_{EF} = \frac{U_0}{\sqrt{2}}$$

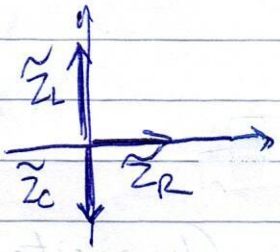
$$\tilde{\mathcal{E}}_0 = (\tilde{Z}_R + \tilde{Z}_L + \tilde{Z}_C) \tilde{I}_0 \Rightarrow \tilde{I}_0 = \frac{\tilde{\mathcal{E}}_0}{\tilde{Z}_R + \tilde{Z}_L + \tilde{Z}_C} =$$

$$= \frac{\mathcal{E}_0}{R + i\left[\omega L - \frac{1}{\omega C}\right]} = \frac{\mathcal{E}_0 \left[R - i\left[\omega L - \frac{1}{\omega C}\right]\right]}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$|\tilde{I}_0|^2 = \tilde{I}_0^* I_0 = \frac{\mathcal{E}_0^2}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$\operatorname{tg} \varphi = \frac{\operatorname{Im} \tilde{I}_0}{\operatorname{Re} I_0} = \frac{\dfrac{-\left(\omega L - \frac{1}{\omega C}\right)}{R}}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

ladíme ω t.ž. I_0 nejvyšší $\Rightarrow \omega L = \frac{1}{\omega C}$
$\Rightarrow \omega_0^2 = \frac{1}{LC}$



Rezonance: $\left(\omega L - \frac{1}{\omega C}\right)^2 = R^2 \qquad \omega_1 L - \frac{1}{\omega_1 C} = R$
$\qquad \omega_2 L - \frac{1}{\omega_2 C} = -R$

$$\Rightarrow (\omega_1+\omega_2)L - \frac{1}{C}\left(\frac{1}{\omega_1}+\frac{1}{\omega_2}\right)=0 \Rightarrow L-\frac{1}{C}\left(\frac{1}{\omega_1\omega_2}\right)=0 \Rightarrow \omega_1\omega_2 = \frac{1}{LC}$$

$$\Rightarrow \left(-\frac{1}{\omega_1^2}+\frac{1}{\omega_2^2}\right)\frac{1}{C} = R\left(\frac{1}{\omega_1}+\frac{1}{\omega_2}\right) \Rightarrow \frac{\omega_1-\omega_2}{\omega_1\omega_2} = RC \Rightarrow \boxed{\Delta\omega = \frac{R}{L}}$$

7. Nestacionární mag. pole

$\vec{E} = \vec{E}(\vec{r}, t)$

$\vec{B} = \vec{B}(\vec{r}, t)$ musíme kompenzovat

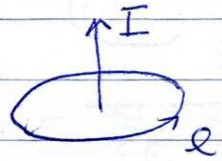
$\rho = \rho(\vec{r}, t)$

$\vdots$

POSUVNÝ PROUD

$$(1)\ \nabla \cdot \vec{j}(\vec{r}, t) + \frac{\partial \rho(\vec{r}, t)}{\partial t} = 0 \quad , \quad \mathrm{I.}\ \frac{dQ}{dt} = 0$$

Amp. z.: $\oint_l \vec{B} \cdot d\vec{l} = \mu_0 I$



$$(2)\ \nabla \times \vec{B}(\vec{r}, t) = \mu_0 \vec{j}(\vec{r}, t)$$ – platí i pro záv. na t!?

dos. (2) do (1): $(4)\ \nabla \cdot \vec{j} = \frac{1}{\mu_0} \nabla \cdot (\nabla \times \vec{B}) = 0$ – divné, něco chybí

Gauss. z.: $\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$

$$\Rightarrow \frac{\partial \rho}{\partial t} = \varepsilon_0 \frac{\partial}{\partial t} \nabla \cdot \vec{E}$$

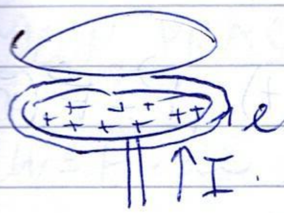
$$(3)\ \frac{\partial \rho}{\partial t} = \nabla \cdot \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

dos. (3) do (1): $\nabla \cdot \vec{j} + \nabla \cdot \varepsilon_0 \frac{\partial \vec{E}}{\partial t} = 0$

$$\nabla \cdot \Big[\vec{j} + \underbrace{\varepsilon_0 \frac{\partial \vec{E}}{\partial t}}_{\text{MAXWELLŮV PROUD}}\Big] = 0$$

$$\boxed{\vec{j}_{MAXW.} := \varepsilon_0 \frac{\partial \vec{E}}{\partial t}}$$

dos. $\vec{j}_{MAXW}$ do (4): $\boxed{\nabla \times \vec{B} = \mu_0 \vec{j} + \mu_0 \vec{j}_{MAXW}}$



$E = \frac{\sigma}{\varepsilon_0}$, $\sigma = \frac{Q}{S}$, $I = \frac{dQ}{dt}$

$\oint_l \vec{B} \cdot d\vec{l} = \mu_0 I$, ale pro všechny plochy;

pro plochu v kond. $I = 0 \Rightarrow \oint = 0$ – rozpor

$I = \dot{Q} = S\dot{\sigma} = \underbrace{S\varepsilon_0 \dot{E}}_{\text{rozměr } j} \Rightarrow j = \boxed{\varepsilon_0 \frac{\partial E}{\partial t} \cdot\ j_{MAXW}}$

pro vakuum:

– změna proudu spočívá v časové změně el. pole.

pro dielektrikum: $D = \sigma \quad \varepsilon_0 \vec{E} + \vec{P}$

$\vec{j}_{\text{„MAXW"}} = \frac{\partial \vec{D}}{\partial t}$

$$\vec{j}_{\text{„MAXW"}} = \boxed{\underbrace{\varepsilon_0 \frac{\partial \vec{E}}{\partial t}}_{=:\ \vec{j}_{POS}} + \frac{\partial \vec{P}}{\partial t}}$$

– POSUVNÝ PROUD

– změna proudu spočívá navíc v časové změně polarizace (viz pružinkový model dipólu)

$\boxed{\vec{j}_{POL} := \frac{\partial \vec{P}}{\partial t}}$ – POLARIZAČNÍ PROUD

zpět k Amp. z.: $\nabla \times \vec{B}(\vec{r}, t) = \mu_0 \vec{j} + \mu_0 \vec{j}_{MAXW} + \mu_0 \vec{j}_{POL} + \mu_0 \vec{j}_{MAG}$

klasický vodivostní   magnetizační proud

opak.: $\vec{j}_{MAG} = \nabla \times \vec{M}$

$$\nabla \times [\vec{B} - \mu_0 \vec{M}] = \mu_0 [\vec{j} + \vec{j}_{POS}]$$

$$\nabla \times \left[\frac{\vec{B}}{\mu_0} - \vec{M}\right] = \vec{j} + \vec{j}_{POS},$$

když $\vec{B}/\mu_0 = \vec{H} + \vec{M}$: $\boxed{\nabla \times \vec{H}(\vec{r}, t) - \frac{\partial \vec{D}}{\partial t} = \vec{j}}$ – analog. k stac mag. $\nabla \times \vec{E}(\vec{r}) - \frac{\partial \vec{B}}{\partial t} = \vec{j}$

MAXWELLOVY ROVNICE

$f = f(\vec{r}, t)$, $\vec{F} = \vec{F}(\vec{r}, t)$

$$\boxed{\nabla \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = \vec{0}}$$

ELMAG. INDUKCE

$$\boxed{\nabla \times \vec{H} - \frac{\partial \vec{D}}{\partial t} = \vec{j}}$$

VYLEPŠENÝ AMPÉREŮV ZÁKON
ZAPOČTENÍ POSUV. PROUDU
ROVNICE KONTINUITY PROUDU

$$\boxed{\nabla \cdot \vec{D} = \rho}$$

GAUSSŮV ZÁKON

$$\boxed{\nabla \cdot \vec{B} = 0}$$

NEEXISTENCE MAG. MONOPÓLŮ

$\updownarrow$

$$\boxed{\oint_l \vec{E} \cdot d\vec{l} = -\frac{d\Phi}{dt}}$$

$$\boxed{\oint_l \vec{H} \cdot d\vec{l} = \int_S \frac{\partial \vec{D}}{\partial t} \cdot \vec{n}\, dS}$$

$$\boxed{\oint \vec{D} \cdot \vec{n}\, dS = Q}$$

$$\boxed{\oint_S \vec{B} \cdot \vec{n}\, dS = 0}$$

- proměnné E, D, H, B; v těch jsou skryty vlastnosti látek – někdy se doplňují tzv. materiálové vztahy

$\vec{B} = \vec{B}(\vec{H})$, $\vec{D} = \vec{D}(\vec{E})$

např. pro vakuum $\vec{B} = \mu_0 \vec{H}$, $\vec{D} = \varepsilon_0 \vec{E}$

- někdy se doplňuje též Ohmův zákon
- tyto rovnice popisují pole, když chceme popisovat pohyby částic přidáme Lorentzovu sílu

$\vec{F} = Q\vec{E} + Q\,\vec{v} \times \vec{B}$

VÝKON, PRÁCE ELMAG. POLE

$$dW = \vec{F}\cdot d\vec{l} = Q\vec{E}\cdot d\vec{l} + Q(\vec{v}\times\vec{B})\cdot d\vec{l}$$

$$\frac{dW}{dt} = Q\vec{E}\cdot\underbrace{\frac{d\vec{l}}{dt}}_{=\vec{v}} + \underbrace{Q(\vec{v}\times\vec{B})\cdot\frac{d\vec{l}}{dt}}_{=0}$$

$$\Rightarrow \frac{dW}{dt} = Q\vec{E}\cdot\vec{v}$$

výkon na objem

$$p_V = \underbrace{\underbrace{N\cdot Q}_{=\rho}\cdot\vec{v}}_{=\vec{j}}\cdot\vec{E} = \vec{j}\cdot\vec{E}$$

dosadím z Maxw.: $\vec{j}\cdot\vec{E} = \left(\nabla\times\vec{H} - \frac{\partial\vec{D}}{\partial t}\right)\cdot\vec{E}$

Pozn.:
velikost j_{MAXW}? $\quad j_{MAXW} = \varepsilon_0\frac{\partial E}{\partial t}$, $\vec{E} = E_0\cos\omega t$

$$j_{MAXW} = -\varepsilon_0\omega E_0\sin\omega t = j_{MAXW_0}\sin\omega t$$

ve vodiči: $j = \frac{1}{\rho_R}E = \frac{1}{\rho_R}E_0\cos\omega t = j_0\cos\omega t$ (ρ_R měr. odp.)

$$\Rightarrow \frac{j_{MAXW}}{j_0} = \varepsilon_0\omega\rho \propto 10^{-18} - 10^{-19} f$$

$$j\cdot\vec{E} = (\nabla\times\vec{H})\cdot\vec{E} - \frac{\partial\vec{D}}{\partial t}\cdot\vec{E}$$

víme: $\nabla\cdot(\vec{E}\times\vec{H}) = \vec{H}\cdot(\nabla\times\vec{E}) - \vec{E}\cdot(\nabla\times\vec{H})$

$$\Rightarrow j\cdot\vec{E} = \vec{H}\cdot(\nabla\times\vec{E}) - \nabla\cdot(\vec{E}\times\vec{H}) - \vec{E}\cdot\frac{\partial\vec{D}}{\partial t}$$

dosadím z Maxw.: $\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$

$$j\cdot\vec{E} = -\nabla\cdot(\vec{E}\times\vec{H}) - \left[\vec{E}\cdot\frac{\partial\vec{D}}{\partial t} + \vec{H}\cdot\frac{\partial\vec{B}}{\partial t}\right]$$

když nyní vztáhneme na objem V

$$P = \frac{dW}{dt} = \int_V \vec{j}\cdot\vec{E}\, dV = -\int_V \nabla\cdot(\vec{E}\times\vec{H})dV - \int_V \left[\vec{E}\cdot\frac{\partial\vec{D}}{\partial t} + \vec{H}\cdot\frac{\partial\vec{B}}{\partial t}\right]dV$$

Gauss. v. int. p.:

$$\boxed{\frac{dW}{dt} = -\oint_{S(V)} (\vec{E}\times\vec{H})\cdot\vec{n}\,dS - \int_V \underbrace{\left[\vec{E}\cdot\frac{\partial\vec{D}}{\partial t} + \vec{H}\cdot\frac{\partial\vec{B}}{\partial t}\right]}_{\text{změna hustoty energie v objemu}} dV}$$

POYNTINGŮV TEORÉM

$$\boxed{\frac{\partial w_E}{\partial t} = \vec{E}\cdot\frac{\partial\vec{D}}{\partial t} + \vec{H}\cdot\frac{\partial\vec{B}}{\partial t}}$$

pro vakuum: $\vec{D} = \varepsilon_0\vec{E}$, $\vec{B} = \mu_0\vec{H}$

$$\frac{\partial w_E}{\partial t} = \vec{E}\cdot\varepsilon_0\frac{\partial\vec{E}}{\partial t} + \vec{H}\cdot\mu_0\frac{\partial\vec{H}}{\partial t} = \frac{\varepsilon_0}{2}\frac{\partial}{\partial t}(\vec{E}\cdot\vec{E}) + \frac{\mu_0}{2}\frac{\partial}{\partial t}(\vec{H}\cdot\vec{H})$$

$$\frac{\partial w_E}{\partial t} = \frac{\partial}{\partial t}\left[\underbrace{\tfrac{1}{2}\vec{E}\cdot\vec{D}}_{} + \underbrace{\tfrac{1}{2}\vec{H}\cdot\vec{B}}_{}\right]$$ – platí i pro konst. ε, μ; obecně toto

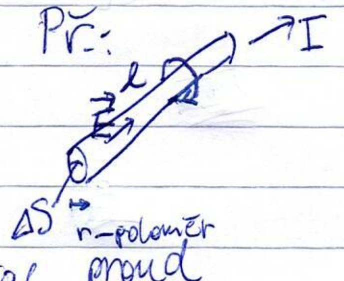
výrazy pro energ. elektr. (kondenz.) a magn. (solenoid) pole

a co plošný sčítanec?

$\vec{S}(\vec{r},t) = \vec{E}(\vec{r},t)\times\vec{H}(\vec{r},t)$ – POYNTINGŮV VEKTOR

– první sčítanec je tok výkonu pde povrchem $S(V)$, přičemž $\vec{S}$ určuje lokální hodnotu tohoto toku (za předpokladu, že je tok rovnoměrný v různých místech – dodnes není jisté, ale asi platí)

Př.:

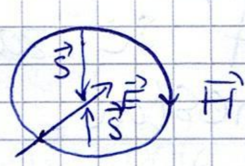


stac. proud

Amp. z. $\oint_l \vec{H}\cdot d\vec{l} = I$

$$2\pi r H = I \longrightarrow H = \frac{1}{2\pi r}I = \frac{I}{2\pi r}$$

Ohmův z.: $j = \frac{1}{\varrho_R} E \Rightarrow E = \varrho_R j = \frac{\varrho_R I}{\Delta S}$



$\Rightarrow S = \frac{\varrho_R}{\Delta S} \frac{1}{2\pi r} I^2,$

když $S_{PLÁŠTĚ} = 2\pi r l$

$TOK = S \cdot S_{PLÁŠTĚ} = \cancel{2\pi r} l \frac{1}{\cancel{2\pi r}} \frac{\varrho_R}{\Delta S} I^2 = RI^2$ – Jouleovské teplo

- výkon je způsoben polem okolo vodiče samotné nosiče se pohybují v běžném obvodu o mikrometry

Př.:

$\oint \vec{H} \cdot d\vec{l} = I$

$2\pi r H = I \rightarrow H = \frac{I}{2\pi r}, \quad E = \frac{\sigma}{\varepsilon_0}$

$\Rightarrow S = \|\vec{E} \times \vec{H}\| = \frac{1}{2\pi r \varepsilon_0} I \underbrace{\sigma}_{= \frac{Q}{S}}$

$S_{PLÁŠTĚ} = 2\pi r d$

$TOK = S_{PLÁŠTĚ} \cdot S = 2\pi r d \cdot \frac{1}{2\pi r \varepsilon_0} I \frac{Q}{S} = \underbrace{\frac{d}{\varepsilon_0} \cdot \frac{1}{S}}_{= 1/C} QI,$

když $I = \frac{dQ}{dt}$: $TOK = \frac{d}{dt}\left(\frac{1}{2}\frac{1}{C}Q^2\right)$

- pole indukuje nábojovou hustotu
- poyntingův vek. funguje i v stac. polích
- do zachování energie může přispívat i mechanika

POTENCIÁLY NESTACIONÁRNÍHO ELMAG. POLE

pro stac. mag. pole: $\vec{B}(\vec{r}) = \nabla \times \vec{A}(\vec{r})$

pro elstat. pole: $\vec{E}(\vec{r}) = -\nabla \varphi(\vec{r})$

$\vec{A}$ se zavádí, neboť $\nabla \cdot \vec{B} = 0$, to platí i v nestac. mag. p.

$\Rightarrow \vec{B}(\vec{r},t) = \nabla \times \vec{A}(\vec{r},t)$

φ se zavádí, neboť $\nabla \times \vec{E} = 0$, to neplatí; platí $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

$$\nabla \times \vec{E} = -\frac{\partial}{\partial t}(\nabla \times \vec{A})$$

$$\nabla \times \vec{E} = -\nabla \times \frac{\partial \vec{A}}{\partial t}$$

$$\nabla \times \underbrace{\left(\vec{E} + \frac{\partial \vec{A}}{\partial t}\right)}_{} = \vec{0}$$

vektor, jehož rotace je nulová - lze zavést skalární potenciál $\boxed{\varphi(\vec{r},t): \vec{E} = -\frac{\partial \vec{A}}{\partial t} - \nabla \varphi(\vec{r},t)}$

- zobecnění φ

a co napojení na zdroje? (řešme ve vakuu)

1. $\nabla \cdot \vec{D} = \rho$, $\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$

$$\nabla \cdot \left(-\frac{\partial \vec{A}}{\partial t} - \nabla \varphi\right) = \frac{\rho}{\varepsilon_0}$$

$$-\frac{\partial}{\partial t}\nabla \cdot \vec{A} - \underbrace{\nabla \cdot \nabla}_{\triangle}\varphi = \frac{\rho}{\varepsilon_0}$$

$$\triangle \varphi + \frac{\partial}{\partial t}(\nabla \cdot \vec{A}) = -\frac{\rho}{\varepsilon_0}$$ - analog. Poissonovy rce

2. $\nabla \times \vec{H} - \frac{\partial \vec{D}}{\partial t} = \vec{j}$

$$\nabla \times \vec{B} - \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} = \mu_0 \vec{j}$$

$$\nabla \times (\nabla \times \vec{A}) + \mu_0 \varepsilon_0 \frac{\partial}{\partial t}\left(\frac{\partial \vec{A}}{\partial t} + \nabla \varphi\right) = \mu_0 \vec{j}$$

$$\nabla \cdot (\nabla \cdot \vec{A}) - \triangle \vec{A} + \mu_0 \varepsilon_0 \frac{\partial^2}{\partial t^2}\vec{A} + \mu_0 \varepsilon_0 \frac{\partial}{\partial t}\nabla \varphi = \mu_0 \vec{j}$$

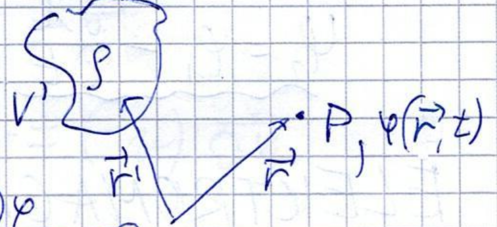
sta

$$\Delta\vec{A} - \mu_0\varepsilon_0\frac{\partial^2}{\partial t^2}\vec{A} - \nabla\left[\nabla\cdot\vec{A} + \mu_0\varepsilon_0\frac{\partial\varphi}{\partial t}\right] = -\mu_0\vec{j}$$

1. Coulombova kalibrace $\vec{A}$: $\nabla\cdot\vec{A} = 0$ – pro stac. pole, ale vhodná i pro nestac, pak

$$\Delta\varphi(\vec{r},t) = -\frac{\rho(\vec{r},t)}{\varepsilon_0} \Rightarrow \underbrace{\varphi(\vec{r},t) = \frac{1}{4\pi\varepsilon_0}\cdot\int_{V'}\frac{\rho(\vec{r}',t)}{|\vec{r}-\vec{r}'|}\,dV'}$$

vypadá $\vec{E}$ stejně jako pro stac.?
ne! ve výpočtu $\vec{E}$ navíc $\frac{\partial A}{\partial t}$



2. Lorenzova kalibrace $\vec{A}$: $\nabla\cdot\vec{A} + \mu_0\varepsilon_0\frac{\partial\varphi}{\partial t} = 0$

$$\Rightarrow \Delta\vec{A} - \mu_0\varepsilon_0\frac{\partial^2\vec{A}}{\partial t^2} = \mu_0\vec{j}$$

$$\Rightarrow \Delta\varphi + \frac{\partial}{\partial t}\underbrace{\nabla\cdot\vec{A}}_{= -\mu_0\varepsilon_0\frac{\partial\varphi}{\partial t}} = -\frac{\rho}{\varepsilon_0}$$

$$\Delta\varphi - \mu_0\varepsilon_0\frac{\partial^2\varphi}{\partial t^2} = -\frac{\rho}{\varepsilon_0}$$

podobné vlnové rce

Λ

KALIBRAČNÍ TRANSFORMACE

$$\vec{A}_2 = \vec{A}_1 + \vec{a}(\vec{r},t)$$
$$\varphi_2 = \varphi_1 + f(\vec{r},t)$$

$$\vec{B} \Rightarrow \nabla\times\vec{A}_1 = \nabla\times\vec{A}_2$$
$$\Rightarrow \nabla\times\vec{A}_1 = \nabla\times\vec{A}_1 + \nabla\times\vec{a}$$
$$\Rightarrow \nabla\times\vec{a} = \vec{0}$$

volme např. $\vec{a}(\vec{r},t) = \nabla\Lambda(\vec{r},t)$

$$-\nabla\varphi_2 - \frac{\partial A_2}{\partial t} = -\nabla\varphi_1 - \frac{\partial A_1}{\partial t} \Leftarrow \vec{E}$$

$$\cancel{-\nabla\varphi_1} - \nabla f - \frac{\partial\vec{a}}{\partial t} - \cancel{\frac{\partial\vec{A}_1}{\partial t}} = \cancel{-\nabla\varphi_1} - \cancel{\frac{\partial\vec{A}_1}{\partial t}}$$

$$\nabla f + \frac{\partial\vec{a}}{\partial t} = 0, \text{ když } \vec{a} = \nabla\Lambda$$

$$\nabla f + \nabla\frac{\partial\Lambda}{\partial t} = \vec{0} \Rightarrow \nabla\left(f + \frac{\partial\Lambda}{\partial t}\right) = 0$$

$$f + \frac{\partial \Lambda}{\partial t} =: \chi(t) \quad \text{— nezáv. na } \vec{r}$$

$$\left.\begin{aligned} f &= \underbrace{\chi(t) - \frac{\partial \Lambda(\vec{r},t)}{\partial t}}_{=: \, -\frac{\partial \lambda(\vec{r},t)}{\partial t}} \end{aligned}\right\} \Rightarrow \frac{\partial \lambda}{\partial t} = -\chi + \frac{\partial \Lambda}{\partial t}$$

$$\Rightarrow \lambda = \int \chi \, dt + \Lambda + C$$

$$\Rightarrow \vec{A}_2 = \vec{A}_1 + \nabla \lambda$$

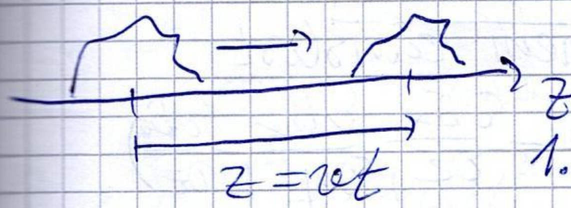
$$\varphi_2 = \varphi_1 - \frac{\partial \lambda}{\partial t} \qquad \lambda = \lambda(\vec{r}, t)$$

ELEKTROMAGNETICKÉ VLNY

8. Elektromagnetické vlny

VLNY

$$f(z,t) = f(\underbrace{z - vt}_{=:\,\xi}) = f(z + vt)$$



1. $$\frac{\partial f}{\partial z} = \frac{\partial f}{\partial \xi} \cdot \frac{\partial \xi}{\partial z} = \frac{\partial f}{\partial \xi}$$

$$\frac{\partial f}{\partial t} = \frac{\partial f}{\partial \xi} \cdot \frac{\partial \xi}{\partial t} = (-v)\frac{\partial f}{\partial \xi}$$

2. $$\left.\begin{aligned} \frac{\partial^2 f}{\partial z^2} &= \frac{\partial^2 f}{\partial \xi^2} \\ \frac{\partial^2 f}{\partial t^2} &= v^2 \frac{\partial^2 f}{\partial \xi^2} \end{aligned}\right\} \frac{\partial^2 f}{\partial z^2} - \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2} = 0 \text{ – VLNOVÁ RCE}$$

– vlna je fce, která splňuje vlnovou rovnici

Maxw. ve vakuu:

$$\left.\begin{aligned} \nabla \times \vec{E} + \frac{\partial \vec{B}}{\partial t} &= \vec{0} \\ \nabla \times \vec{H} - \frac{\partial \vec{D}}{\partial t} &= \vec{0} \end{aligned}\right)$$

$$\nabla \times (\nabla \times \vec{E}) + \nabla \times \frac{\partial \vec{B}}{\partial t} = \vec{0}$$

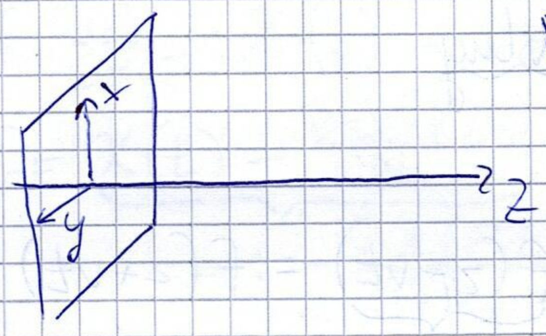
$$\nabla(\underbrace{\nabla \cdot \vec{E}}_{=0}) - \Delta \vec{E} + \frac{\partial}{\partial t}(\nabla \times \mu_0 \vec{H}) = \vec{0}$$

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0},\ \rho = 0 \Rightarrow$$

$$-\Delta \vec{E} + \mu_0 \frac{\partial}{\partial t}\left(\frac{\partial \vec{D}}{\partial t}\right) = \vec{0}$$

$$\boxed{\Delta \vec{E} - \varepsilon_0 \mu_0 \frac{\partial^2 \vec{E}}{\partial t^2} = \vec{0}}$$

$$\boxed{\Delta \vec{B} - \varepsilon_0 \mu_0 \frac{\partial^2 \vec{B}}{\partial t^2} = \vec{0}}$$



$$\nabla\cdot\vec{E} \underset{\rho=0}{=} 0 \Rightarrow \underbrace{\frac{\partial E_x}{\partial x}}_{=0} + \underbrace{\frac{\partial E_y}{\partial y}}_{=0} + \underbrace{\frac{\partial E_z}{z}}_{\Rightarrow\, =0} = 0$$

$$\Rightarrow \boxed{E_z = \text{konst.}}$$ – PŘÍČNÁ VLNA

$$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} = -\left(\frac{\partial E_y}{\partial z} - \frac{\partial E_z}{\partial y}, \frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z}, \frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial z}\right)$$

$$\left(-\frac{\partial E_y}{\partial z}, \frac{\partial E_x}{\partial z}, 0\right) = -\left(\frac{\partial B_x}{\partial t}, \frac{\partial B_y}{\partial t}, 0\right)$$

$$\frac{\partial E_y}{\partial z} = \frac{\partial B_x}{\partial t} \qquad \frac{\partial E_x}{\partial z} = -\frac{\partial B_y}{\partial t}$$

$$\frac{\partial E_y}{\partial z} = -v\frac{\partial B_x}{\partial z} \qquad \frac{\partial E_x}{\partial z} = v\frac{\partial B_y}{\partial z}$$

$$\boxed{E_y = -vB_x} \qquad \boxed{E_x = vB_y}$$

$\vec{E}\cdot\vec{B} = \ldots = 0$ – charakteristické pro příčné vlny

$(\vec{B} \perp \vec{E})$

$$\vec{E}\times\vec{B} = (E_yB_z - E_zB_y, E_zB_x - E_xB_z, E_xB_y - E_yB_x)$$

$$= (0, 0, vB_x^2 + vB_y^2) \Rightarrow B_x^2 + B_y^2 = B^2 \quad (B_z = 0)$$

$$= (0, 0, vB^2)$$

$$\mu_0 \underbrace{(\vec{E}\times\vec{H})}_{\text{Poynting. v.}} = (0, 0, vB^2)$$

$\hookrightarrow \vec{S} = \left(0, 0, \frac{v}{\mu_0}B^2\right)$ – energie se šíří ve směru vlny

$$B^2 = B_x^2 + B_y^2 = \frac{1}{v^2}E_y^2 + \frac{1}{v^2}E_x^2 = \frac{1}{v^2}E^2$$

$$\vec{S} = \left(0, 0, \frac{1}{\mu_0 v}E^2\right)$$

St

z toho vyplývá $\frac{1}{v^2} = \mu_0 \varepsilon_0$ pro elmag.
ve vakuu => rychlost světla

$w_E = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2}\mu_0 H^2$

$\left(\underbrace{\frac{1}{2}\vec{E}\cdot\vec{D}}_{w_{ELE}} + \underbrace{\frac{1}{2}\vec{H}\cdot\vec{B}}_{w_{MAG}}\right)$

$S_2 = \frac{v}{\mu_0}B^2 = v\mu_0 H^2$, $\quad S_2 = \frac{1}{v\mu_0}E^2$

$= 2w_{MAG}\cdot v \qquad\qquad = \frac{1}{v\mu_0}2w_{ELE}\cdot\frac{1}{\varepsilon_0} = 2w_{ELE}\cdot v$

=> $w_{MAG} = w_{ELE}$ v rovinné vlně

=> $S_2 = v\cdot w_E$ (analog. $j = v\cdot\rho$)

0)